

第四屆培正數學邀請賽
4th Pui Ching Invitational Mathematics Competition

決賽（中一組）
Final Event (Secondary 1)

時限：2 小時

Time allowed: 2 hours

參賽者須知：

Instructions to Contestants:

1. 本卷共設甲、乙兩部分，總分爲 100 分。
This paper is divided into Section A and Section B. The total score is 100.
2. 除特別指明外，本卷內的所有數均爲十進制。
Unless otherwise stated, all numbers in this paper are in decimal system.
3. 除特別指明外，所有答案須以數字的真確值表達，並化至最簡。不接受近似值。
Unless otherwise stated, all answers should be given in exact numerals in their simplest form.
No approximation is accepted.
4. 把所有答案填在答題紙指定的空位上。毋須呈交計算步驟。
Put your answers on the spaces provided on the answer sheet. You are not required to hand in your steps of working.
5. 不得使用計算機。
The use of calculators is not allowed.
6. 本卷的附圖不一定依比例繪成。
The diagrams in this paper are not necessarily drawn to scale.

甲部 (60分)

Section A (60 marks)

第 1 至第 4 題，每題 3 分。

Questions 1 to 4 each carries 3 marks.

第 5 至第 8 題，每題 5 分。

Questions 5 to 8 each carries 5 marks.

第 9 至第 12 題，每題 7 分。

Questions 9 to 12 each carries 7 marks.

1. 一張大小為 16×24 的長方形紙張可以剪成 n 小塊，使得每小塊皆是正方形。（每小塊的大小不一定相同。）求 n 的最小可能值。

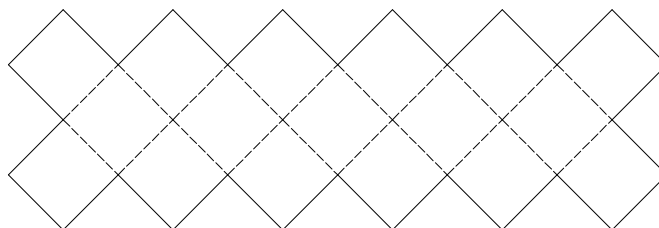
A rectangular piece of paper of size 16×24 can be cut into n small pieces such that each small piece is square in shape. (The small pieces are not necessarily of the same size.) Find the smallest possible value of n .

2. 甲、乙二人參加吃熱狗大賽。甲每 7 秒吃一隻熱狗，乙每 11 秒吃一隻熱狗。兩人前面都有一個顯示器，顯示兩人已吃完的熱狗數目。甲的顯示器所顯示的數字首次比乙的顯示器所顯示的數字大 77 時是開賽後多少秒？

A and B join a hot-dog eating competition. A eats a hot-dog every 7 seconds while B eats a hot-dog every 11 seconds. There is a counter in front of each person showing the number of hot-dogs they have eaten. How many seconds have lapsed from the beginning of the competition when the counter of A exceeds that of B by 77 for the first time?

3. 下圖是一種特別的棋盤。該棋盤由 17 個面積為 1 的正方形組成。這種棋的棋子都是 1×2 的長方形，而且必須放在棋盤上兩個相鄰的方格內。如果棋子不容許重疊，那麼棋盤上最多可放多少隻棋子？

The figure below shows a special type of chessboard. The chessboard is made up of 17 squares of area 1. The chess pieces are all 1×2 rectangles, and must be placed on two consecutive squares of the chessboard. If overlapping is not allowed, what is the maximum number of pieces of chess that can be placed on the chessboard?

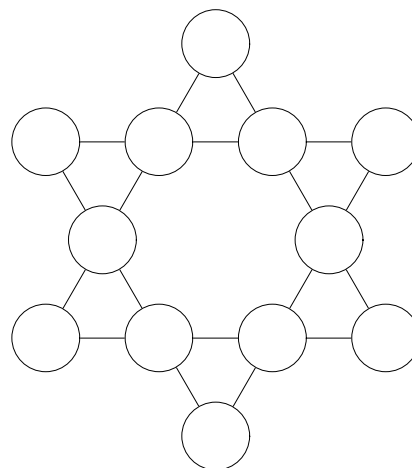


4. 圖中所示的是一個幻方，它每個橫行、每個直行和每條對角線上三個數之和都是相同的。然而，幻方中有些方格留空了。左下角那一格的數應該是甚麼？

13	4	
		8
		9

The figure shows a magic square in which the sum of the three numbers in each of its rows, columns and diagonals is the same. However, some entries are missing. What should be the number in the lower left hand corner?

5. 小芬在圖中的每個圓圈內分別填上 1 至 12 之間的其中一個整數，並且沒有重覆。她發現，任何四個成一直線的圓圈內的四數之和都不同。這些和之中，設最大的一個為 M ，最小的一個為 m 。求 $M - m$ 的最小值。

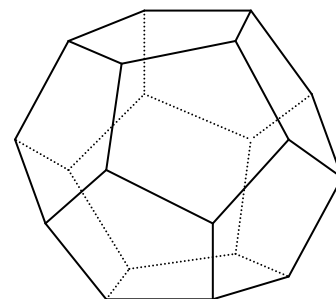


Sally put one of the integers from 1 to 12 into each of the circles in the figure without repetition. She found that the sum of any four numbers which lie on the same straight line is different from each other. Let M and m be the largest and smallest of these sums, respectively. Find the smallest value of $M - m$.

6. 三所學校被邀請出席一個大型活動。主辦單位安排了旅遊車接載參加活動的師生，而每輛旅遊車的載客量都是相同的。甲校有 53 名師生出席，需要 4 輛旅遊車；乙校有 113 名師生出席，需要 7 輛旅遊車。若丙校有 2005 名師生出席，需要多少輛旅遊車？

Three schools are invited to attend a mass function. The organizers have arranged coaches to send students and teachers to the venue. All coaches carry the same number of passengers. There are 53 participants in School A and 4 coaches are needed. There are 113 participants in School B and 7 coaches are needed. If there are 2005 participants in School C, how many coaches are needed for School C?

7. 一隻螞蟻在一個正十二面體的其中一個頂點出發，沿著它的邊走。如果該螞蟻不走重覆的路，且最後回到它的出發點，那麼該螞蟻最多可走過多少條邊？



On a regular dodecahedron, an ant starts at a vertex and walks along its edges. If the ant does not repeat its path and eventually returns to the starting point, what is the maximum number of edges that the ant can go through?

8. 在所示的算式中，不同的字母代表不同的數字。已知 $S = 2$ 、 $T = 4$ ，求四位數 MATH 的值。

In the calculation shown, different letters represent different digits. Given that $S = 2$ and $T = 4$, find the value of the four-digit integer MATH.

$$\begin{array}{r}
 P \quad C \quad M \quad S \\
 M \quad A \quad T \quad H \\
 S \quad A \quad T \\
 + 9 \\
 \hline
 A \quad P \quad R \quad I \quad L
 \end{array}$$

9. 小華和小明玩遊戲：現有兩個袋子，它們分別有 1200 枚和 1800 枚硬幣。兩人按以下的規則輪流取去硬幣：

- 每人每次只可在其中一個袋子裡取硬幣，最少取走 1 枚硬幣，但不可把袋中所有硬幣取走。
- 每次取完硬幣後，必須有至少一個袋子中的硬幣數量是偶數。

若某人不能按上述規則取走硬幣便算落敗，另一人則勝出。如果小華先取，他第一次要取多少枚硬幣才可確保自己勝出？

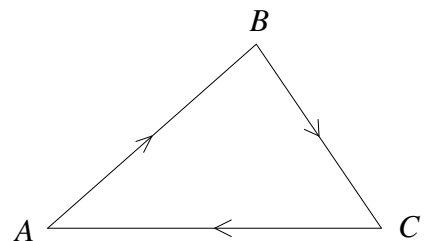
Roy plays a game with Sam as follows. There are 2 bags, with 1200 and 1800 coins respectively. They take away coins from the bags in turns according to the following rules:

- One may take away coins from only one of the bags each time. At least 1 coin must be taken away and one must not take away all the coins in the bag.
- After each round, there must be at least one bag with an even number of coins.

Whoever fails to take away coins according to the above rules is the loser while the other player is the winner. If Roy takes away coins first, how many coins should he take away in the first round in order to guarantee that he must win?

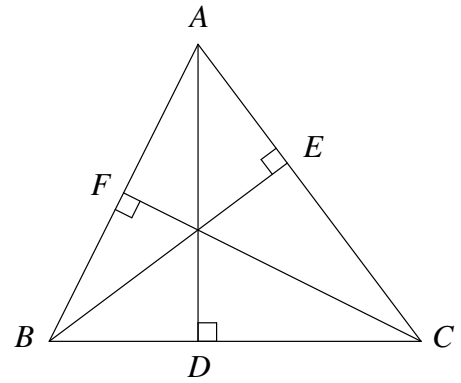
10. 圖中， $AB = 25$ m、 $BC = 20$ m、 $CA = 30$ m。小美、比蒂和嘉兒分別從 A 點、 B 點和 C 點出發，沿著三角形的邊依順時針方向分別以 x m/s、 1.5 m/s 和 2 m/s 的速率散步。若其中兩人相遇，則她們會結伴同行，而步行速度以較慢者為準。若她們首次三人相遇的地方是 B 點，求 x 的最小值。

In the figure, $AB = 25$ m, $BC = 20$ m and $CA = 30$ m. Amy, Betty and Cathy start at A , B and C respectively, walking along the sides of the triangle in the clockwise direction, with speeds x m/s, 1.5 m/s and 2 m/s respectively. When two of them meet, they will walk together at the lower speed among them. If the first time the three girls meet is at B , find the minimum value of x .



11. 在 $\triangle ABC$ 中， $AB = 4$ 、 $BC = 7$ 、 $CA = 5$ 。 AD 、 BE 、 CF 是 $\triangle ABC$ 的三條垂線。求 $\frac{AD+BE}{CF}$ 的值。

In $\triangle ABC$, $AB = 4$, $BC = 7$ and $CA = 5$. AD , BE and CF are the three altitudes of $\triangle ABC$. Find the value of $\frac{AD+BE}{CF}$.



12. 黃先生和三個女孩子嘉嘉、麗麗和敏敏玩遊戲。他有 7 張分別寫上了 1、2、3、4、5、6 和 7 的咭片。黃先生向每位女孩子派了兩張咭，並藏起了餘下的一張咭。每人原來只可以看到自己手上的咭，但後來黃先生要求每位女孩子把自己的一張咭給另一名女孩子看。於是，麗麗給嘉嘉看自己的一張咭，敏敏給麗麗看自己的一張咭，嘉嘉給敏敏看自己的一張咭。以下是他們於較後時間的對話。

黃先生問：「有人知道其他女孩子的咭上的數字嗎？」

嘉嘉說：「不知道，但我知道麗麗的數字之和是單數。」

敏敏說：「聽過嘉嘉的說話後，我仍不知道她們二人的數字是甚麼。但我知道我的數字之和比嘉嘉的數字之和的大。」

麗麗說：「我知道嘉嘉和敏敏的數字了。敏敏的數字之和比我的數字之和的小呢。」

假設所有女孩子都是聰明的（即當她們有足夠的資料便一定可以作出推論），求麗麗的數字之積。

Mr Wong played a game with 3 girls Karen, Lucy and Mary. There are 7 cards with numbers 1, 2, 3, 4, 5, 6 and 7 respectively. Mr Wong gave 2 cards to each girl and hid the remaining card. Originally, every girl could only see her own cards. Later, Mr Wong asked each girl to let another girl see one of her cards. As a result, Lucy let Karen see one of her cards, Mary let Lucy see one of her cards and Karen let Mary see one of her cards. Their subsequent conversations are as follows.

“Does anyone know the numbers on the cards of the other girls?” asked Mr Wong.

“No, but I know that the sum of the numbers on Lucy’s cards is odd.” replied Karen.

“After listening to Karen, I still do not know what their numbers are. But I know that the sum of my numbers is greater than that of Karen’s.” replied Mary.

“I know Karen’s and Mary’s numbers now,” said Lucy, “the sum of Mary’s numbers is smaller than that of mine.”

Assume that all girls are intelligent (i.e. they can make deductions whenever there is enough information). What is the product of the Lucy’s numbers?

乙部 (40 分)

Section B (40 marks)

13. 某人用以下方法寫出一個數列：首項是「2」，然後是「1」；接著是「3」，然後有兩個「1」；之後是「4」，然後是三個「1」，如此類推。

$$2, 1, 3, 1, 1, 4, 1, 1, 1, \dots$$

Someone forms a sequence as follows. The first term is '2', followed by a '1'. Next comes a '3', followed by two '1's. After that there is a '4', followed by three '1's, and so on.

$$2, 1, 3, 1, 1, 4, 1, 1, 1, \dots$$

- (a) 已知數列的第 n 項是 2005。求 n 。 (3 分)
Given that the n -th term of the sequence is 2005, find n . (3 marks)
- (b) 數列的首 2005 項中，有多少項不是 1？ (4 分)
How many of the first 2005 terms of the sequence are not 1? (4 marks)

如果我們把以上數列的項累積地加起來，我們可得到另一個新數列

$$2, 3, 6, 7, 8, 12, 13, 14, 15, \dots$$

例如，因為原數列中首六項之和是 $2+1+3+1+1+4=12$ ，所以新數列的第六項是 12。

If we cumulatively sum up the terms of the above sequence, we can get another sequence

$$2, 3, 6, 7, 8, 12, 13, 14, 15, \dots$$

For instance, since the sum of the first six terms of the original sequence is $2+1+3+1+1+4=12$, the sixth term of the new sequence is 12.

- (c) 在新數列所有小於 10000 的項當中，有多少個是 3 的倍數？ (6 分)
Among all the terms of the new sequence which are smaller than 10000, how many are divisible by 3? (6 marks)
- (d) 志文在新數列中選了 30 個連續項，並準備了 30 根木棒，木棒的長度（以 mm 為單位）剛好等於那 30 個連續項。他發現其中三根木棒不能砌成三角形。若 30 根木棒中最長的一根的長度為 x mm，求 x 的最大可能值。 (7 分)
Jacky picks 30 consecutive terms of the new sequence and prepares 30 wooden sticks with lengths (in mm) equal to these 30 terms. He finds that three of the wooden sticks cannot form a triangle. If the longest of the 30 wooden sticks has length x mm, find the greatest possible value of x . (7 marks)

14. 本屆培正數學邀請賽中，除個人獎項外，亦會頒發學校團體獎。為比較參賽者在不同卷別中的表現，我們會先把各參賽者的分數化成「相對得分」，方法如下：

$$\text{相對得分} = \text{學生的實際得分} \div \text{同級參賽者中首八名的平均得分}$$

在初賽中，各級的試卷皆有 20 題，其中 3 分題、4 分題、5 分題、6 分題和 7 分題各佔 4 題，滿分為 100 分。每所學校最多可派 8 名學生參賽，中一至中四級各 2 名。我們以下式計算「學校的初賽得分」：

$$\text{學校的初賽得分} = \text{該校學生在初賽的相對得分總和} \div 8$$

In the current Pui Ching Invitational Mathematics Competition, an overall award for schools will be given in addition to individual awards. In order to compare the performance of participants in different papers, the score of each participant will first be converted into a 'relative score' by the following formula:

$$\text{Relative Score} = \text{Actual Score} \div \text{Average score of the 8 best students in the same paper}$$

In the Heat Events, each paper consists of 20 questions, including four of each of 3-mark, 4-mark, 5-mark, 6-mark and 7-mark questions. The full score is 100. Each school may send at most 8 students to participate, 2 for each of Secondary 1 to Secondary 4. We get a 'School Score for Heats' according to the following formula:

$$\text{School Score for Heats} = \text{Sum of all relative scores of its students in the Heat Events} \div 8$$

註：理論上，在計算相對得分的公式中，除數可能等於 0，使相對得分變得沒有意義。然而，只有當某級所有參賽者皆得 0 分時，這個情況才會發生，而這在實際情況中是極不可能的。因此，在制訂比賽章則時，我們已假設了該除數不等於 0。同學回答此題時，亦應作同樣的假設。

Remark. Theoretically, in the formula for computing the relative score, the divisor can be equal to 0 which makes the relative score undefined. However, this will not happen unless all participants in a certain form get 0 mark. This is extremely unlikely in reality. Hence, when enacting the details of the competition, we have assumed that the above denominator will not be equal to 0. Contestants should make the same assumption when answering this question.

- (a) 若某名中一級的參賽者在初賽得到 50 分，他的相對得分最低是多少？ (3 分)

If a Secondary 1 student gets 50 marks in the Heat Event, what is the minimum relative score he may get? (3 marks)

- (b) 在初賽中一組的試卷中，「同級參賽者中首八名的平均得分」有多少個不同的可能值？（注意我們已假設了此數不等於 0。） (4 分)

How many different possible values of the 'average score of the 8 best students in the same paper' are there for the Secondary 1 paper of the Heat Event? (Note that we have assumed that this is not equal to 0.) (4 marks)

- (c) 學校的初賽得分的最大可能值是多少？ (6 分)

What is the maximum School Score for Heats that a school may attain? (6 marks)

- (d) 學校的初賽得分可以寫成分數。假設某校的初賽得分約至最簡後為 $\frac{p}{q}$ ，且 k 是 q 的一個質因數，求 k 的最大可能值。 (7 分)

The School Score for Heats can be written as a fraction. Suppose that the School Score for Heats for a certain school is $\frac{p}{q}$ in lowest terms. If k is a prime factor of q , find the greatest possible value of k . (7 marks)

全卷完

END OF PAPER