

第一屆香港培正中學數學邀請賽

1st Pui Ching Middle School Invitational Mathematics Competition

個人賽 (中二組)

Individual Event (Secondary 2)

模擬試題

Sample Paper

時限：1 小時

Time allowed: 1 hour

參賽者須知：

Instructions to Contestants:

1. 本卷共設 20 題，總分為 100 分。

There are 20 questions in this paper and the total score is 100.

2. 除特別指明外，本卷內的所有數均為十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

3. 除特別指明外，所有答案須以數字的真確值表達，並化至最簡。不接受近似值。

Unless otherwise stated, all answers should be given in exact numerals in their simplest form.
No approximation is accepted.

4. 所有答案填在答題紙指定的空位上。毋須呈交計算步驟。

Put your answers on the spaces provided on the answer sheet. You are not required to hand in your steps of working.

5. 不得使用計算機。

The use of calculators is not allowed.

6. 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

第 1 至第 5 題，每題 2 分。

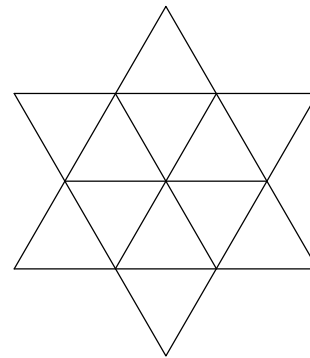
Questions 1 to 5, each carries 2 marks.

1. 某日，由於交通擠塞，一輛公共汽車的平均速率降低了 20%。那麼，行車時間增加了百分之幾？

Because of traffic congestion, the average speed of a bus was reduced by 20% one day. By what percentage was the traveling time increased?

2. 圖中有多少個三角形？

How many triangles are there in the figure?



3. n 為一正整數，若 n 的各位數字之和等於 10，則 n 稱為「好數」。若把所有「好數」從小至大排列，則第 15 個「好數」是甚麼？

A positive integer n is said to be 'good' if the sum of the digits of n is equal to 10. If all the 'good' numbers are arranged in ascending order, what is the 15th 'good' number?

4. 求 1122、2233 和 3344 的最大公因數。

Find the highest common factor (H.C.F.) of 1122, 2233 and 3344.

5. 求最小的正整數 n ，使得 n 的所有質因數均大於 90。

Find the least positive integer n such that every prime factor of n is greater than 90.

第 6 至第 10 題，每題 4 分。

Questions 6 to 10 each carries 4 marks.

6. 從昨天上午 3 時到今天上午 3 時之間，時鐘上的時針和分針共重疊了多少次？

Between 3:00 am yesterday and 3:00 am today, how many times did the minute hand and hour hand of a clock overlap?

7. $6^{54} \times 3^{21}$ 的個位數字是甚麼？

What is the unit digit of $6^{54} \times 3^{21}$?

8. 一正整數 n 除以 3 時，餘數為 2；除以 5 時，餘數為 4；除以 7 時，餘數為 6。求 n 的最小可能值。

A positive integer n leaves a remainder of 2 when divided by 3, 4 when divided by 5 and 6 when divided by 7. Find the smallest possible value of n .

9. 在一個 3×3 的表格中，每格分別填上 1 至 9 的整數，使得每格的數字均不同，且每行、每列及兩條對角線上的數字之和均等於 k 。求 k 。

A 3×3 table is filled with the integers from 1 to 9 so that each cell has a different entry. Also, the sums of the numbers in each row, each column and each of the two diagonals are all equal to k . Find k .

10. 某正多邊形的每隻內角為 179.99° 。這多邊形有多少條邊？

Each interior angle of a regular polygon is 179.99° . How many sides does this polygon have?

第 11 至第 15 題，每題 6 分。

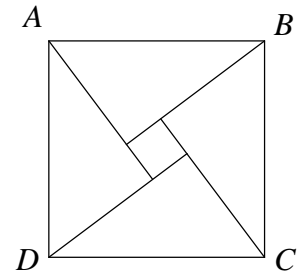
Questions 11 to 15, each carries 6 marks.

11. 求 $1^{2001} + 2^{2001} + 3^{2001} + \dots + 2000^{2001} + 2001^{2001}$ 的個位數字。

Find the unit digit of $1^{2001} + 2^{2001} + 3^{2001} + \dots + 2000^{2001} + 2001^{2001}$.

12. 如圖所示，四個相同的直角三角形的兩條較短的邊分別是 20 和 21。現把它們拼成一個正方形 $ABCD$ ，求 AB 。

In the figure, four identical right-angled triangles, of which the shorter sides are 20 and 21, are combined to form a square $ABCD$. Find AB .



13. 紙上畫了 22 個圓形。它們最多共有多少個交點？

22 circles are drawn on a piece of paper. What is the maximum number of intersections that these 22 circles can make?

14. 已知對所有正整數 n ， $\frac{1}{n} - \frac{1}{n+2} = \frac{2}{n(n+2)}$ 。求 $\frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \dots + \frac{1}{2001 \times 2003}$ 的值。

It is known that for all positive integers n , $\frac{1}{n} - \frac{1}{n+2} = \frac{2}{n(n+2)}$. Find the value of

$$\frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \dots + \frac{1}{2001 \times 2003}.$$

15. 某人以時速 40 公里駕車由甲地駛往乙地，再以時速 60 公里由乙地沿原路駛回甲地。若全程的平均速率為 x km/h，求 x 。

A man drove from City A to City B at a speed of 40 km/h and then drove back from City B to City A via the same route at a speed of 60 km/h. If the average speed of the whole journey was x km/h, find x .

第 16 至第 20 題，每題 8 分。

Questions 16 to 20, each carries 8 marks.

16. 在以下算式中，每個字母代表一個由 0 至 9 的數字，但不同字母所代表的數字可能相同。求 A、B、C、D、E、F、G 各字母所代表的數字之總和。

In the following calculation, each letter represents a digit from 0 to 9, but different letters do not necessarily represent different digits. Find the sum of the numbers represented by the letters A, B, C, D, E, F and G.

$$\begin{array}{rcccccccc} & A & B & C & D & E & F & G \\ & & B & C & D & E & F & G \\ & & & C & D & E & F & G \\ & & & & D & E & F & G \\ & & & & & E & F & G \\ & & & & & & F & G \\ + & & & & & & & G \\ \hline & B & B & B & B & B & B & B \end{array}$$

17. 定義 $a \oplus b = ab + a + b$ 。求以下數式的值：

$$(((2002 \oplus 2003) \oplus 2004) \oplus 2005) - (((2005 \oplus 2004) \oplus 2003) \oplus 2002)$$

Define $a \oplus b = ab + a + b$. Find the value of the following expression:

$$(((2002 \oplus 2003) \oplus 2004) \oplus 2005) - (((2005 \oplus 2004) \oplus 2003) \oplus 2002)$$

18. 設今天為星期六，則 $2^{2002} + 2002 + k$ 天後亦是星期六，當中 k 是正整數。若 k 的各位數字之和為完全平方數（例如：1、4、9、16、...），求 k 的最小可能值。

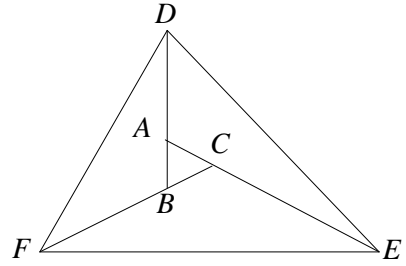
Suppose that today is Saturday and it will also be a Saturday after $2^{2002} + 2002 + k$ days. If the sum of digits in k is a perfect square (e.g. 1, 4, 9, 16, ...), find the smallest possible value of k .

19. 在長方形 $ABCD$ 內有一點 P 。如果 $AP = 5$ ， $CP = 5$ ， $DP = 1$ ，求 BP 的長度。

There is a point P in a rectangle $ABCD$. If $AP = 5$, $CP = 5$, $DP = 1$, find the length of BP .

20. 如圖所示， A 、 B 、 C 為三角形 DEF 內的點，使得 DAB 、 FBC 、 ECA 為直線， $AB = AD$ ， $CE = 3AC$ ， $2FB = 5BC$ 。若三角形 ABC 的面積為 1，求三角形 DEF 的面積是多少？

In the figure, A , B , C are points inside triangle DEF such that DAB , FBC and ECA are straight lines, $AB = AD$, $CE = 3AC$ and $2FB = 5BC$. If the area of triangle ABC is 1, what is the area of triangle DEF ?



全卷完

END OF PAPER

答案：

Answers:

1. 25%

2. 20

3. 154

4. 11

5. 97

6. 22

7. 8

8. 104

9. 15

10. 36000

11. 1

12. 29

13. 462

14. $\frac{1001}{2003}$

15. 48

16. 43

17. 0

18. 40

19. 7

20. 20.5