第十屆培正數學邀請賽

10th Pui Ching Invitational Mathematics Competition

初賽（高中組）

Heat Event (Senior Secondary)

時限：1 小時 15 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

(a) 本卷共設 20 題，總分爲 100 分。

There are 20 questions in this paper and the total score is 100.

(b) 除特別指明外，本卷內的所有數均爲十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

(c) 所有答案皆是 0 至 9999 之間的整數（包括 0 和 9999）。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

All answers are integers between 0 and 9999 (including 0 and 9999). Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

(d) 不得使用計算機。

The use of calculators is not allowed.

(e) 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

1. 若 x 是正數，且 $(x^3-2011x)^2=0$ ，求最接近 x 的整數。 （3分）

If x is a positive number such that $(x^3-2011x)^2=0$, find the integer closest to x. (3 marks)

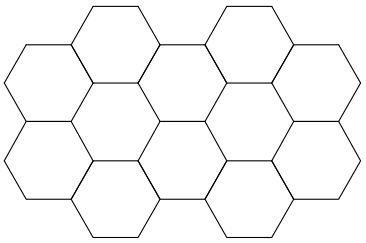
2. 若 $\log_{2011}1+\log_{2011}2+\cdots+\log_{2011}n>1$ ，求 n 的最小可能值。 （3分）

If $\log_{2011}1+\log_{2011}2+\cdots+\log_{2011}n>1$, find the smallest possible value of n. (3 marks)

3. 在一個等差數列中，若第 9 項比第 5 項大 60，則第 2011 項比第 2000 項大多少？ （3分）

In an arithmetic sequence, the 9th term is greater than the 5th term by 60. By how much is the 2011th term greater than the 2000th term? (3 marks)

4. 詩敏把圖中 12 個小六邊形的其中 n 個塗上顏色，並發現沒有兩個塗了色的六邊形有公共邊。求 n 的最大可能值。 （3分）



Angel coloured n of the 12 small hexagons in the figure and found that no two coloured hexagons have a common side. Find the greatest possible value of n. (3 marks)

5. 若把一個球體切成兩個半球體，則其總表面積會增加 n%。求最接近 n 的整數。 （3分）

If a sphere is cut into two hemispheres, the total surface area will increase by n%. Find the integer closest to n. (3 marks)

6. 求 C_3^{2011} 的個位數字。 （4分）

Find the unit digit of C_3^{2011}. (4 marks)

7. 設 $[x]$ 代表不超過 x 的最大整數，例如 [1.1] = 1、[6.9] = 6 和 [5] = 5。求 $[\tan 2011°]$ 的值。 （4分）

Let $[x]$ denote the greatest integer not exceeding x. For example, [1.1] = 1, [6.9] = 6 and [5] = 5. Find the value of $[\tan 2011°]$. (4 marks)

8. 某次考試後，老師決定給每位同學加 10 分。那麼，在分數的平均數、中位數、眾數、方差、標準差、分佈域、最大值、第 60 百分位數、上四分位數和四分位數間距這 10 項數據中，增加 10 分的有多少項？ （4 分）

After an examination, the teacher decided to add 10 marks to each student. Among the following 10 items of the scores: mean, median, mode, variance, standard deviation, range, maximum, 60th percentile, upper quartile and interquartile range, how many will increase by 10 marks? (4 marks)

9. 設 b、c 爲常數。若 x^2+bx+c 除以 $x+2$ 和 $x+4$ 時的餘數相同，求 b。 （4 分）

Let b and c be constants. If x^2+bx+c leaves the same remainder when divided by $x+2$ and $x+4$, find b. (4 marks)

10. 某圓的方程是 $kx(2x+y)-4y(5x-hy)=2011$，其中 k、h 是常數。求 $k+h$ 的值。 （5 分）

A circle has equation $kx(2x+y)-4y(5x-hy)=2011$, where k and h are constants. Find the value of $k+h$. (5 marks)

11. 求坐標平面上所有滿足不等式 $|x-2|+|y-3|\le 4$ 的點 (x, y) 所形成的區域的面積。 （5 分）

Find the area of the region formed by points (x, y) on the coordinate plane which satisfy the inequality $|x-2|+|y-3|\le 4$. (5 marks)

12. 某國家共有六種不同面值的硬幣。如果每種硬幣各取一枚，則在不設找贖的情況下共可繳付 n 個不同的正數金額。求 n 的最小可能值。 （5 分）

In a country, there are six types of coins with different denominations. By having one of each type of coins, a total of n different positive amounts can be paid without change. Find the smallest possible value of n. (5 marks)

13. 如果三位數 $\overline{abc}$ 可被 5 整除，同時方程 $ax^2+bx+c=0$ 沒有實根，那麼我們說 $\overline{abc}$ 是「壞數」。「壞數」共有多少個？ （6 分）

A three-digit number $\overline{abc}$ is said to be ‘bad’ if it is divisible by 5 and the equation $ax^2+bx+c=0$ has no real root. How many ‘bad’ numbers are there? (6 marks)

14. 在某等角八邊形中，各邊的長度依次是 9、10、11、12、13、14、p、q。求 q，答案準確至最接近整數。（6分）

In an equiangular octagon, the side lengths are 9, 10, 11, 12, 13, 14, p, q in order. Find q correct to the nearest integer. (6 marks)

15. 穎淇不斷投擲一顆公平的骰子，直至連續兩次擲得偶數便停止。若她擲得偶數的總次數是兩次的概率是 n%，求最接近 n 的整數。（7分）

Vicky keeps throwing a fair die until two even numbers are obtained in a row. If the probability that she has obtained an even number exactly twice is n%, find the integer closest to n. (7 marks)

16. 求 $\int_{\frac{\pi}{20}}^{\frac{\pi}{10}} \frac{100\cos 5x}{1-\cos 10x}\,\mathrm{d}x$ 的值，答案準確至最接近整數。（7分）

Find the value of $\int_{\frac{\pi}{20}}^{\frac{\pi}{10}} \frac{100\cos 5x}{1-\cos 10x}\,\mathrm{d}x$ correct to the nearest integer. (7 marks)

17. 若某正整數除最左數字外，每個數字都比它左邊的數字大，則稱爲「好數」，例如：123 和 24689 都是「好數」。在所示的乘式中，每個字母代表一個由 0 至 9 的不同數字，而被乘數 ABC 和乘積 PQRST 都是「好數」。求積的最後四位數字（即 QRST）。（7分）

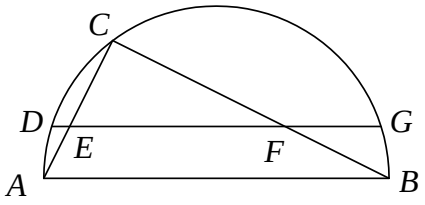
$$\begin{array}{rr} & A\ B\ C \\ \times & B\ C \\ \hline P\ Q\ R\ S\ T & \end{array}$$

A positive integer is said to be ‘good’ if each of its digits (except the leftmost one) is greater than the digit on its left. For instance, 123 and 24689 are ‘good’. In the multiplication shown, each letter represents a different digit from 0 to 9, and the multiplicand ABC and the product PQRST are both ‘good’. Find the last four digits of the product (i.e. QRST). (7 marks)

18. 一個邊長爲 3 的正方體被分成 27 個邊長爲 1 的小正方體。現要選取三個小正方體，使它們的中心成一直線，問共有多少種不同的方法？（7分）

A cube of side length 3 is divided into 27 small cubes of side length 1. Now three small cubes are to be selected so that their centres lie on the same straight line. How many different choices are there? (7 marks)

19. 圖中，*AB* 是半圓的直徑，*C* 是圓周上的一點。直線 *DEFG* 與 *AB* 平行，其中 *D*、*G* 位於圓周上，*E*、*F* 分別位於 *AC* 和 *BC* 上。若 $AC = 30$ 而 $BC = EF = 40$，求 *DG* 的長度，答案準確至最接近整數。（7 分）



In the figure, *AB* is the diameter of the semi-circle and *C* is a point on the circumference. The straight line *DEFG* is parallel to *AB*, with *D*, *G* lying on the circumference and *E*, *F* lying on *AC* and *BC* respectively. If $AC = 30$ and $BC = EF = 40$, find the length of *DG* correct to the nearest integer. (7 marks)

20. 一張咭紙上有 10 個圓，分別編號爲 0 至 9。現要把一個或多個圓填色，並規定如果編號爲 n 的圓填了色，則編號爲 n^2 的個位數字的圓亦必須填色。求填色方法的總數。（7 分）

On a piece of cardboard there are 10 circles, numbered 0 to 9. We shall now colour one or more circles subject to the following condition: if the circle numbered n is coloured, then the circle whose number is the unit digit of n^2 must also be coloured. Find the number of ways of colouring. (7 marks)

全卷完

END OF PAPER