第十屆培正數學邀請賽

10th Pui Ching Invitational Mathematics Competition

初賽（中三組）

Heat Event (Secondary 3)

時限：1 小時 15 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

(a) 本卷共設 20 題，總分爲 100 分。

There are 20 questions in this paper and the total score is 100.

(b) 除特別指明外，本卷內的所有數均爲十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

(c) 所有答案皆是 0 至 9999 之間的整數（包括 0 和 9999）。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

All answers are integers between 0 and 9999 (including 0 and 9999). Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

(d) 不得使用計算機。

The use of calculators is not allowed.

(e) 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

1. 求 $1+2+3+\cdots+2011$ 的個位數字。 （3分）

Find the unit digit of $1+2+3+\cdots+2011$. (3 marks)

2. 在一個梯形中，其中三隻內角分別是 60°、70° 和 x°。求 x 所有可能值之和。 （3分）

In a trapezium, three interior angles are 60°, 70° and x°. Find the sum of all possible values of x. (3 marks)

3. 首 2011 個正整數（即 1、2、…、2011）的平均值是多少？ （3分）

What is the mean of the first 2011 positive integers (i.e. 1, 2, …, 2011)? (3 marks)

4. 首 2011 個正整數中，可被 3 整除但不可被 4 整除的有多少個？ （4分）

How many of the first 2011 positive integers are divisible by 3 but not by 4? (4 marks)

5. 設 A 和 B 爲兩個三角柱體，而 A 的一個面和 B 的一個面全等。若把 A 和 B 的這兩個面黏合起來形成一個新的立體，則新立體最少有幾個面？ （4分）

Let A and B be two triangular prisms. One face of A is congruent to one face of B. If these two faces of A and B are glued together to form a new solid, what is the minimum number of faces of the new solid? (4 marks)

6. 乘積 $4\times51\times601\times7001\times80001\times900001$ 共有多少個數字？ （4分）

How many digits are there in the product $4\times51\times601\times7001\times80001\times900001$? (4 marks)

7. 某七位數可被 2011 整除。若它的首三位是 246，則它的最後四位的最小值是甚麼？ （4分）

A seven-digit number is divisible by 2011. If its first three digits are 246, what is the smallest possible value of its last four digits? (4 marks)

8. 某奇數的數字之和是 2011。若把該數除以 18，餘數是多少？ （5分）

The sum of digits of an odd number is 2011. If the number is divided by 18, what is the remainder? (5 marks)

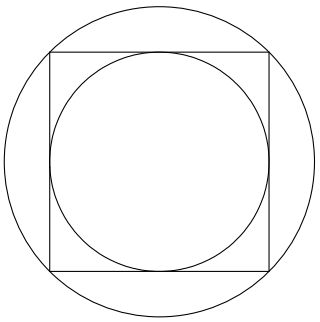
9. 已知 $2011\times129=259419$ 和 $129^2=16641$。在乘積 20110129×20110129 中有多少個「4」字？ （5分）

Given $2011\times129=259419$ and $129^2=16641$. How many '4's are there in the product 20110129×20110129? (5 marks)

10. 在某次考試中，老師決定調整各人的分數，方法是按平日表現給予每位學生一個獎分。如果一名學生的考試分數是 E 而獎分是 B，則其調整後的分數爲 $5B+\frac{E}{100}(100-5B)$。珮儀和凱欣都有參加考試，雖然兩人的考試分數不同，但調整後的分數卻相同。若珮儀在考試中取得 100 分，則凱欣所得的獎分是多少？ （5分）

In an examination, the teacher decided to adjust the scores by awarding a bonus score to each student depending on his/her daily performance. For a student with examination score E and bonus score B, his/her adjusted score will be $5B+\frac{E}{100}(100-5B)$. Polly and Queenie took the examination and, despite having different examination scores, ended up with the same adjusted score. If Polly got a score of 100 in the examination, what bonus score was Queenie awarded? (5 marks)

11. 圖中，小圓的面積是大圓的面積的 n%。求最接近 n 的整數。 （5分）



In the figure, the area of the smaller circle is n% that of the larger circle. Find the integer closest to n. (5 marks)

12. 在某個正 2011 邊形中，每個頂點與中心的距離是 1。求該多邊形的面積，答案準確至最接近整數。 （5分）

In a regular 2011-sided polygon, the distance from each vertex to its centre is 1. Find the area of the polygon correct to the nearest integer. (5 marks)

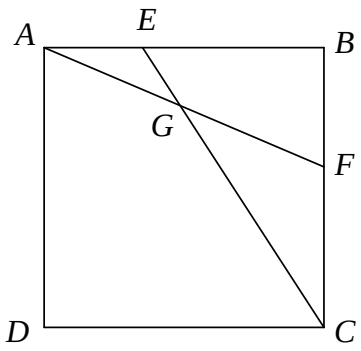
13. 求最小的正整數 n，使得 $2011n$ 的最後四位數字是 0129。 （6分）

Find the smallest positive integer n so that the last four digits of $2011n$ are 0129. (6 marks)

14. 某班有 40 名學生，老師叫每人寫下一個正整數。老師其後發現，共有 n 種方法選兩名學生，使得其中一人寫下的正整數是另一人的兩倍。求 n 的最大可能值。（6 分）

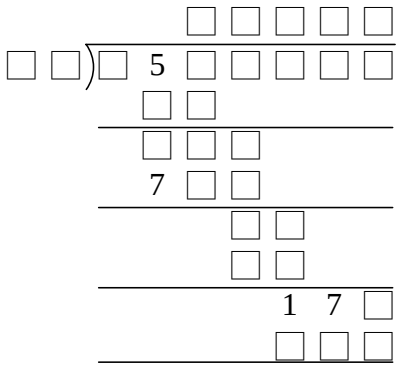
There are 40 students in a class. The teacher asked each student to write down a positive integer, and subsequently found that there were n ways to choose two students such that the positive integer written by one of them is twice that of the other. Find the largest possible value of n. (6 marks)

15. 圖中，$ABCD$ 是面積爲 10000 的正方形。E 和 F 分別是 AB 和 BC 上的點，使得 $AE:EB=1:2$ 和 $BF:FC=3:4$。AF 交 CE 於 G。求四邊形 $BFGE$ 的面積，答案準確至最接近整數。（6 分）



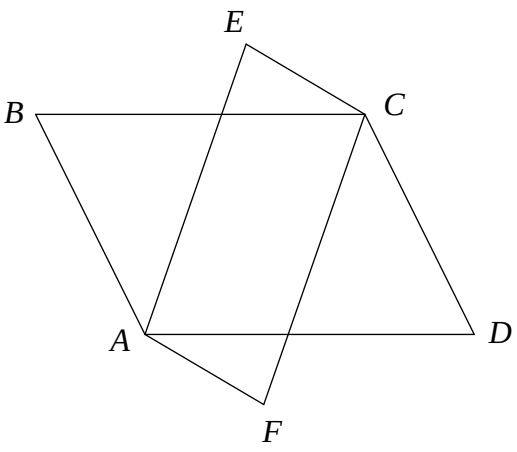
In the figure, $ABCD$ is a square of area 10000. E and F are points on AB and BC respectively such that $AE:EB=1:2$ and $BF:FC=3:4$. AF and CE meet at G. Find the area of quadrilateral $BFGE$ correct to the nearest integer. (6 marks)

16. 圖中顯示一條除式，但當中有些數字留空了。求商（即最頂一行）的最後四位數字。（6 分）



The figure shows a division, but some digits are left out. Find the last four digits of the quotient (i.e. the top row). (6 marks)

17. 圖中，$ABCD$ 和 $AECF$ 是平行四邊形，其中 $AC=AD=CF$。若 $\angle BCF = 48°$ 而 $\angle ECD = x°$，求 x。（6 分）



In the figure, $ABCD$ and $AECF$ are parallelograms and $AC=AD=CF$. If $\angle BCF = 48°$ and $\angle ECD = x°$, find x. (6 marks)

18. 某國家共有六種硬幣，面值分別是 1 元、2 元、3 元、10 元、20 元和 50 元。如果每種硬幣各取一枚，則在不設找贖的情況下共可繳付多少個不同的正數金額？ （6 分）

In a country, there are six types of coins, with denominations $1, $2, $3, $10, $20 and $50 respectively. By having one of each type of coins, how many different positive amounts can be paid without change? (6 marks)

19. 在坐標平面上，一隻螞蟻要從 (0, 0) 前往 (6, 6)。若每步只能上移或右移一單位，並且不能經過 (3, 3)，則共有多少條不同的路線？ （7 分）

On the coordinate plane, an ant wants to travel from (0, 0) to (6, 6). In each step the ant may only travel one unit upward or rightward, and it must not pass through the point (3, 3). How many different routes are possible? (7 marks)

20. 在一個直角三角形中，斜邊比另外兩邊的長度之和小 52。若三角形最短的一條高的長度爲 60，求三角形最短邊的長度。 （7 分）

In a right-angled triangle, the hypotenuse is 52 shorter than the sum of the other two sides. If the shortest altitude has length 60, find the length of the shortest side. (7 marks)

全卷完

END OF PAPER