初賽（中二組）

**Heat Event (Secondary 2)**

時限：**1** 小時 **15** 分

**Time allowed: 1 hour 15 minutes**

參賽者須知：

**Instructions to Contestants:**

(a)  本卷共設 20 題，總分爲 100 分。

There are 20 questions in this paper and the total score is 100.

(b)  除特別指明外，本卷內的所有數均爲十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

(c)  所有答案皆是 0 至 9999 之間的整數（包括 0 和 9999）。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

All answers are integers between 0 and 9999 (including 0 and 9999). Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

(d)  不得使用計算機。

The use of calculators is not allowed.

(e)  本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

1.  某數學競賽首屆有 30 所學校參加，之後每屆的參賽學校數目都比前一年多 22 所。那麼該數學競賽的第十屆有多少所學校參加？ （3分）

    A mathematical competition attracted a total of 30 participating schools in the first year, and in each subsequent year the number of participating schools was 22 more than that in the previous year. How many schools participated in the 10th competition? (3 marks)

2.  已知 2011 可寫成 $k$ 個質數之和。求 $k$ 的最大可能值。 （3分）

    Given that 2011 can be expressed as the sum of $k$ prime numbers, find the greatest possible value of $k$. (3 marks)

3.  某 2011 個 $n$ 位正整數之和是 20110129。求 $n$。 （3分）

    The sum of 2011 $n$-digit positive integers is 20110129. Find $n$. (3 marks)

4.  求一個三位數 $n$，使得當 $n$ 的數字左右倒轉時，所得的三位數大於 $8n$。 （3分）

    Find a three-digit number $n$ such that, when the digits of $n$ are reversed, the resulting three-digit number is greater than $8n$. (3 marks)

5.  若把一個正方體切成 1000 個全等的小正方體，則 1000 個小正方體的總表面積是原正方體的多少倍？ （3分）

    If a cube is dissected into 1000 identical small cubes, how many times the surface area of the original cube is the total surface area of the 1000 small cubes? (3 marks)

6.  有多少個平方數（包括 0）小於 2011 且可被 8 整除？ （4分）

    How many square numbers (including 0) are less than 2011 and divisible by 8? (4 marks)

7.  某書的各頁以 1、2、…、$n$ 編碼。樂生發現該書有連續兩頁的頁數的數字之和都可被 4 整除。求 $n$ 的最小可能值。 （4分）

    The pages of a book are numbered 1, 2, …, $n$. Jason found that in two consecutive pages of the book, the sum of digits of each of the page numbers is divisible by 4. Find the smallest possible value of $n$. (4 marks)

8. 當 $\underbrace{999\ldots99}_{\text{2011個「9」}}$ 除以 7 時，餘數是多少？ （5分）

 What is the remainder when $\underbrace{999\ldots99}_{\text{2011 digits}}$ is divided by 7? (5 marks)

9. $6^{2011}$ 有多少個奇數的正因數？ （5分）

 How many odd positive factors does $6^{2011}$ have? (5 marks)

10. 從數列 1, 2, 3, 4, … 出發，若把每項 $n$ 以 $n, n+1, n+2, n+3, n+4$ 五項取代，可得到以下的新數列：

   1, 2, 3, 4, 5, 2, 3, 4, 5, 6, 3, 4, 5, 6, 7, 4, 5, 6, 7, 8, …

 新數列的第 2011 項是甚麼？ （5分）

 By starting with the sequence 1, 2, 3, 4, …, and replacing each term $n$ by five terms $n, n+1, n+2, n+3, n+4$, the following new sequence is obtained:

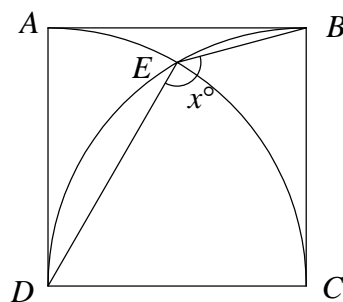   1, 2, 3, 4, 5, 2, 3, 4, 5, 6, 3, 4, 5, 6, 7, 4, 5, 6, 7, 8, …

 What is the 2011th term of the new sequence? (5 marks)

11. 某袋子中有 2011 個球，每個都是紅色或綠色的。每次從袋子中抽出 10 個球時，都總有最少 7 個綠球。袋子中最少有幾個綠球？ （5分）

 There are 2011 balls in a bag, each of which is either red or green. It is known that whenever 10 balls are drawn from the bag, at least 7 must be green. What is the least number of green balls in the bag? (5 marks)

12. 圖中，*ABCD* 是邊長為 1 的正方形。分別以 *C*、*D* 為圓心，作半徑為 1 的弧，使兩弧交於 *E*。若 $\angle BED = x°$，求 $x$。 （6分）

 In the figure, *ABCD* is a square of side length 1. Two arcs, centred at *C* and *D* respectively and each with radius 1, are constructed so that they meet at *E*. If $\angle BED = x°$, find $x$. (6 marks)

13. 設 [*x*] 代表不超過 *x* 的最大整數，例如 [1.1] = 1、[6.9] = 6 和 [5] = 5。求下式 （6分）
的值：

Let [*x*] denote the greatest integer not exceeding *x*. For example, [1.1] = 1, [6.9] = 6 and [5] = 5. Find the value of the following expression: (6 marks)

$$\left[\frac{100000}{2011}\right] - \left[\frac{99999}{2011}\right] + \left[\frac{99998}{2011}\right] - \left[\frac{99997}{2011}\right] + \cdots - \left[\frac{1}{2011}\right]$$

14. 在以下方程組中求 *e*： （6分）

Find *e* in the following system of equations: (6 marks)

$$\begin{cases} a - b + c = 98 \\ b - c + d = 76 \\ c - d + e = 54 \\ d - e + a = 32 \\ e - a + b = 10 \end{cases}$$

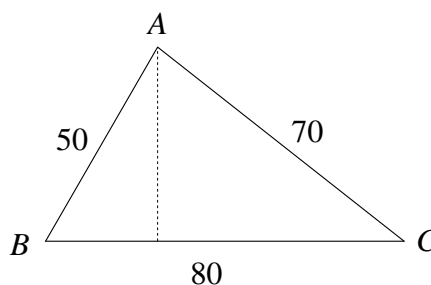15. 某梯形各邊的長度分別是 6、12、12、12。求該梯形的面積，答案準確至最接近整數。 （6分）

The four sides of a trapezium have lengths 6, 12, 12, 12. Find the area of the trapezium correct to the nearest integer. (6 marks)

16. 小莉寫下了 1 到 2011 的正整數，並隨意選了 0 至 9 之間的一個數字 *k*。然後小莉數算在這些正整數中的各位數字中 *k* 出現的次數，得到的結果爲 *N*。求 *N* 的最小可能值。 （6分）

Lily wrote down the positive integers from 1 to 2011, and then randomly chose a digit *k* between 0 and 9. She then counted the number of times *k* has appeared in the digits of the integers, and the result was *N*. Find the smallest possible value of *N*. (6 marks)

17. 在 Δ*ABC* 中，*AB* = 50、*BC* = 80、*CA* = 70。求 *A* 到 *BC* 的高的長度，答案準確至最接近整數。 （6分）

In Δ*ABC*, *AB* = 50, *BC* = 80 and *CA* = 70. Find the length of the altitude from *A* to *BC* correct to the nearest integer. (6 marks)

18. 在所示的乘式中，每個字母代表一個由 0 至 9 的不同數字。求乘積的最後四位數字（即 BACA）。 （7分）

   In the multiplication shown, each letter represents a different digit from 0 to 9. Find the last four digits of the product (i.e. BACA). (7 marks)

$$\begin{array}{r} A\ B\ A \\ \times\quad\ B\ A \\ \hline C\ B\ A\ C\ A \end{array}$$

19. 有多少個九位正整數由三個「1」、三個「2」和三個「3」組成，且沒有兩個連續的「1」字？ （7分）

   How many nine-digit positive integers consist of three '1's, three '2's and three '3's, and have no two consecutive '1's? (7 marks)

20. 某數學競賽的試卷設有 3 分題和 5 分題，每題答對可得該題的分數，否則該題得 0 分。嘉雯和她的妹妹都有參賽，回家後她告訴媽媽：「我和妹妹在競賽中的得分相同，可是我答對的 5 分題比妹妹多」，然後告訴媽媽兩人的得分。媽媽說：「我知道你們各自答對了多少 3 分題和 5 分題了！」求嘉雯的得分的最大可能值。 （7分）

   The paper in a mathematical competition consists of 3-mark questions and 5-mark questions. A correct answer merits all the marks allocated to the question; otherwise 0 mark is given. Both Carmen and her sister joined the competition, and upon returning home she told her mother, 'I got the same score as sister in the competition, but I got more 5-mark questions correct than she did'. She then told her mother their scores. 'I know how many 3-mark questions and 5-mark questions each of you has correctly answered,' said Mum. Find the greatest possible score of Carmen. (7 marks)

全卷完

**END OF PAPER**