第十屆培正數學邀請賽

10th Pui Ching Invitational Mathematics Competition

初賽（中一組）

Heat Event (Secondary 1)

時限：1 小時 15 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

(a) 本卷共設 20 題，總分爲 100 分。

There are 20 questions in this paper and the total score is 100.

(b) 除特別指明外，本卷內的所有數均爲十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

(c) 所有答案皆是 0 至 9999 之間的整數（包括 0 和 9999）。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

All answers are integers between 0 and 9999 (including 0 and 9999). Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

(d) 不得使用計算機。

The use of calculators is not allowed.

(e) 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

1. 某四個 n 位正整數之和是 20110129。求 n。 （3分）

The sum of four n-digit positive integers is 20110129. Find n. (3 marks)

2. 在四邊形 $ABCD$ 中，$\angle A = \angle B$，且 $\angle C = \angle D$。若 $\angle B + \angle C = x°$ ，求 x。 （3分）

In quadrilateral $ABCD$, $\angle A = \angle B$ and $\angle C = \angle D$. If $\angle B + \angle C = x°$, find x. (3 marks)

3. 若把一個正方體切成八個全等的小正方體，則八個小正方體的總表面積是原正方體的多少倍？ （3分）

If a cube is dissected into eight identical small cubes, how many times the surface area of the original cube is the total surface area of the eight small cubes? (3 marks)

4. 有多少個平方數（包括 0）小於 2011 且可被 7 整除？ （4分）

How many square numbers (including 0) are less than 2011 and divisible by 7? (4 marks)

5. 3^{2011} 有多少個正因數？ （4分）

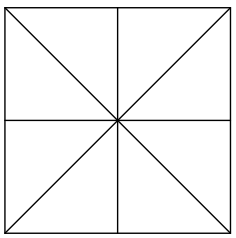
How many positive factors does 3^{2011} have? (4 marks)

6. 在一家商店，顧客可以付 75 元成爲貴賓會員，購物便可享九折價惠。並非該店會員的小美打算在該店購買一件標價 n 元的貨品，她可以選擇以原價購買或成爲貴賓會員以享有九折價惠。她發現不論她成爲會員與否，所需付的總金額都是相同的。求 n。 （4分）

In a shop, a customer can become a VIP member to enjoy 10% discount on all purchases by paying 75 dollars. Mimi, who is not a VIP member of the shop, plans to buy an article from the shop which is marked at n dollars. She can choose to buy it at the marked price or become a VIP member to enjoy the 10% discount. She finds that the total amount she has to pay is the same regardless of her choice. Find n. (4 marks)

7. 在圖中可找到多少個三角形？ （4分）

How many triangles can be found in the figure? (4 marks)



8. 某正方形的一條對角線長度爲 8。它的面積是多少？ （4 分）

A diagonal of a square has length 8. What is its area? (4 marks)

9. 小明沿著一條小徑散步，從起點計每走了 10 步便放下一塊餅。他的小狗於小明放下第 n 塊餅時走到小徑的起點，以小明步速的兩倍沿小徑走，並把沿途的餅吃掉。當小明放下第 60 塊餅時，他發現小狗剛吃掉第 n 塊餅。求 n 的值。 （5 分）

Michael jogged along a path. He put down a piece of biscuit after every 10 steps from the starting point. When Michael put down the n-th biscuit, his dog reached the starting point of the path and walked along at twice the speed of Michael and ate up all the biscuits on the way. When Michael put down the 60th biscuit, he found that his dog had just eaten the n-th biscuit. Find n. (5 marks)

10. 某數學競賽的試卷設有 3 分題和 5 分題，每題答對可得該題的分數，否則該題得 0 分。一名參賽者答對了 12 題，並發現如果 3 分題變成 2 分題，同時 5 分題變成 7 分題的話，他的分數也不會改變。求他的分數。 （5 分）

The paper in a mathematical competition consists of 3-mark questions and 5-mark questions. A correct answer merits all the marks allocated to the question; otherwise 0 mark is given. A contestant answered 12 questions correctly, and found that if the 3-mark questions became 2-mark questions, while the 5-mark questions became 7-mark questions, his score would remain unchanged. Find his score. (5 marks)

11. 從數列 1, 2, 3, 4, … 出發，若把每項 n 以 n, $n+1$, $n+2$, $n+3$, $n+4$ 五項取代，可得到以下的新數列：

1, 2, 3, 4, 5, 2, 3, 4, 5, 6, 3, 4, 5, 6, 7, 4, 5, 6, 7, 8, …

新數列的第 2011 項是甚麼？ （5 分）

By starting with the sequence 1, 2, 3, 4, …, and replacing each term n by five terms n, $n+1$, $n+2$, $n+3$, $n+4$, the following new sequence is obtained:

1, 2, 3, 4, 5, 2, 3, 4, 5, 6, 3, 4, 5, 6, 7, 4, 5, 6, 7, 8, …

What is the 2011th term of the new sequence? (5 marks)

12. 若 x、y、z 滿足以下方程組，求 xyz 的值。 （5 分）

If x, y, z satisfy the following system of equations, find the value of xyz. (5 marks)

$$\begin{cases} x+y=z+2 \\ y+z=x+4 \\ z+x=y+6 \end{cases}$$

13. 一班朋友前往午膳，各人平分賬單。若每人付 16 元，則還欠 5 元；若每人付 17 元，則多出了 17 元。問賬單的費用是多少元？ （6 分）

A group of friends went for lunch and they shared the bill equally. There would be a shortage of $5 if each one paid $16, while there would be an excess of $17 if each one paid $17. How many dollars was the bill? (6 marks)

14. 設 $[x]$ 代表不超過 x 的最大整數，例如 $[1.1] = 1$、$[6.9] = 6$ 和 $[5] = 5$。求下式的值： （6 分）

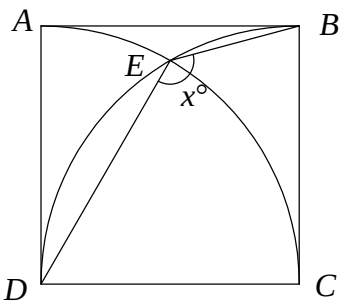
Let $[x]$ denote the greatest integer not exceeding x. For example, $[1.1] = 1$, $[6.9] = 6$ and $[5] = 5$. Find the value of the following expression: (6 marks)

$$\left[\frac{100000}{2011}\right]-\left[\frac{99999}{2011}\right]+\left[\frac{99998}{2011}\right]-\left[\frac{99997}{2011}\right]+\cdots-\left[\frac{1}{2011}\right]$$

15. 小莉寫下了 1 到 2011 的正整數，並隨意選了 0 至 9 之間的一個數字 k。然後小莉數算在這些正整數中的各位數字中 k 出現的次數，得到的結果爲 N。求 N 的最大可能值。 （6 分）

Lily wrote down the positive integers from 1 to 2011, and then randomly chose a digit k between 0 and 9. She then counted the number of times k has appeared in the digits of the integers, and the result was N. Find the greatest possible value of N. (6 marks)

16. 圖中，$ABCD$ 是邊長爲 1 的正方形。分別以 C、D 爲圓心，作半徑爲 1 的弧，使兩弧交於 E。若 $\angle BED = x°$，求 x。 （6 分）



In the figure, $ABCD$ is a square of side length 1. Two arcs, centred at C and D respectively and each with radius 1, are constructed so that they meet at E. If $\angle BED = x°$, find x. (6 marks)

17. 當 $\underbrace{20112011\ldots2011}_{\text{2011 個「2011」}}$ 除以 7 時，餘數是多少？ （6 分）

What is the remainder when $\underbrace{20112011\ldots2011}_{\text{2011 copies of '2011'}}$ is divided by 7? (6 marks)

18. 在所示的乘式中，每個字母代表一個由 0 至 9 的不同數字。求乘積的最後四位數字（即 BACA）。 （7 分）

In the multiplication shown, each letter represents a different digit from 0 to 9. Find the last four digits of the product (i.e. BACA). (7 marks)

$$\begin{array}{rr} & A\ B\ A \\ \times & B\ A \\ \hline C\ B\ A\ C\ A & \end{array}$$

19. 老師寫下了一個小於 100 的正整數 n，然後四位同學分別猜測 n 的性質：

小平：「n 是質數。」

小安：「n 是合成數。」

小健：「n 是平方數。」

小康：「n 是 3 的倍數。」

已知四人當中，兩人猜對了而兩人猜錯了。問 n 有多少個不同的可能值？ （7 分）

The teacher wrote down a positive integer n less than 100. Four students subsequently made guesses on the properties of n:

Aaron guessed 'n is a prime number'.

Bill guessed 'n is a composite number'.

Carson guessed 'n is a square number'.

Dick guessed 'n is a multiple of 3'.

Given that two of the four guesses were correct while two were wrong, how many different possible values of n are there? (7 marks)

20. 有多少個九位正整數由三個「1」、三個「2」和三個「3」組成，且沒有兩個連續的「1」字？ （7 分）

How many nine-digit positive integers consist of three '1's, three '2's and three '3's, and have no two consecutive '1's? (7 marks)

全卷完

END OF PAPER