第九屆培正數學邀請賽

9th Pui Ching Invitational Mathematics Competition

初賽（中四組）

Heat Event (Secondary 4)

時限：1 小時 15 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

(a) 本卷共設 20 題，總分爲 100 分。

There are 20 questions in this paper and the total score is 100.

(b) 除特別指明外，本卷內的所有數均爲十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

(c) 所有答案皆是 0 至 9999 之間的整數（包括 0 和 9999）。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

All answers are integers between 0 and 9999 (including 0 and 9999). Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

(d) 不得使用計算機。

The use of calculators is not allowed.

(e) 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

1. 一個長方形各邊的長度都是整數，而其面積爲 2010。那麼，它的周界的最小可能值是多少？ （3 分）

A rectangle has integral side lengths and area 2010. What is the smallest possible value of its perimeter? (3 marks)

2. 在首 2010 個正整數中，有多少個的數字之和是奇數？ （3 分）

How many of the first 2010 positive integers have an odd sum of digits? (3 marks)

3. 已知 n 個不同正整數之積是 2010。求 n 的最大可能值。 （3 分）

The product of n different positive integers is 2010. Find the greatest possible value of n. (3 marks)

4. 詠彤把 n 個個位數字爲 7 的正整數加起來，得到 2010。求 n 的最小可能值。 （3 分）

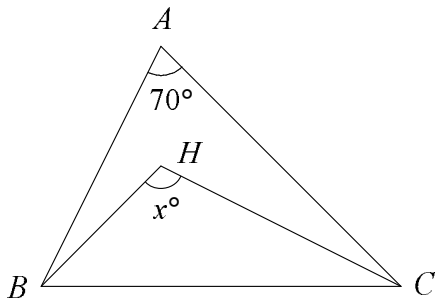
Ada added n positive integers, each with unit digit 7, and obtained a sum of 2010. Find the smallest possible value of n. (3 marks)

5. 求 $y=\frac{1}{6}x$ 、 $y=-x$ 和 $y=-10$ 三條直線所圍出的三角形的面積。 （3 分）

Find the area of the triangle bounded by the straight lines $y=\frac{1}{6}x$, $y=-x$ and $y=-10$. (3 marks)

6. 圖中，H 是 ΔABC 的垂心。求 x。 （4 分）

In the figure, H is the orthocentre of ΔABC. Find x. (4 marks)



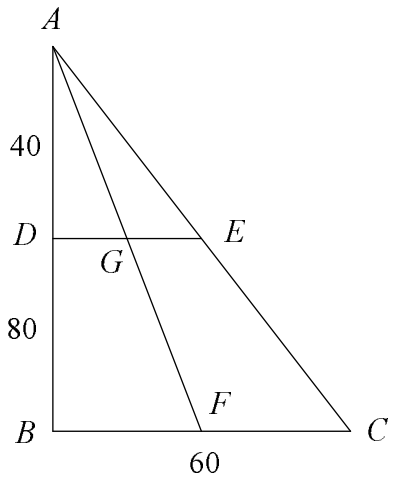
7. 設 $f(x)=(x-2)(x-10)$ 。希彤計算了 $f(-100)$ 、 $f(-99)$ 、 $f(-98)$ 、…、 $f(98)$ 、 $f(99)$ 和 $f(100)$ 的值。她得到多少個不同的數值？ （5 分）

Let $f(x)=(x-2)(x-10)$. Ivy computed the values of $f(-100)$, $f(-99)$, $f(-98)$, …, $f(98)$, $f(99)$ and $f(100)$. How many different values did she obtain? (5 marks)

8. 小權打算寫一個兩位數「27」，怎料個位數字 7 寫得太高，結果變成了 2^7，比原來的兩位數大，小權因此把 27 稱爲「好數」。一般來說，如果把兩位正整數 $\overline{ab}$ 變成 a^b 後得到一個較大的數，那麼便稱 $\overline{ab}$ 爲「好數」。那麼，「好數」共有多少個？ （5 分）

Donald plans to write down the two-digit number 27, but the unit digit 7 was displaced upward and the number becomes 2^7 which is larger than the original two-digit number. Donald therefore calls 27 a 'good' integer. In general, if changing a two-digit positive integer $\overline{ab}$ to a^b yields a larger number, then $\overline{ab}$ is said to be 'good'. How many 'good' numbers are there? (5 marks)

9. 圖中，ABC 是直角三角形，B 是直角。D 和 E 分別是 AB 和 AC 上的點，使得 $AD \perp DE$。F 和 G 分別是 BC 和 DE 的中點。若 $AD = 40$、$DB = 80$ 而 $BC = 60$，求四邊形 $CFGE$ 的面積。 （5 分）

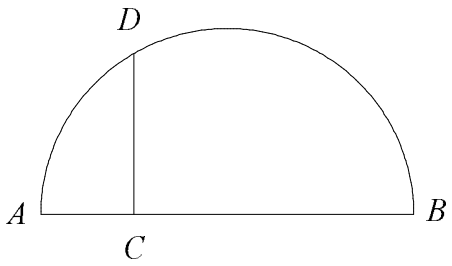


In the figure, ΔABC is right-angled at B. D and E are points on AB and AC respectively such that $AD \perp DE$. F and G are the mid-points of BC and DE respectively. If $AD = 40$, $DB = 80$ and $BC = 60$, find the area of the quadrilateral $CFGE$. (5 marks)

10. 某城市的計程車車費如下：不超過 2 公里的車程一律收 n 元；如果車程超過 2 公里但不超過 10 公里，則超過 2 公里的部分每 200 米收 5 元（不足 200 米亦作 200 米計算，下同）；之後超過 10 公里的部分每 200 米收 6 元。小安發現，雖然長途車程的後部分車費較高，卻不可能在某地方下車再上車來節省車費。求 n 的最小可能值。 （5 分）

In a city the taxi fares are as follows. The minimum fare for journeys not exceeding 2 km is $\$n$. For journeys exceeding 2 km but not exceeding 10 km, \$5 is charged for every 200 m exceeding the first 2 km (portions less than 200 m will be rounded up, same for below). After that \$6 is charged for every 200 m for portions of the journey exceeding 10 km. Andy finds that, despite a higher fare for the latter portion of a long journey, it is not possible to get off the taxi and get on again at some point in order to save money. Find the smallest possible value of n. (5 marks)

11. 圖中顯示一個半徑爲 20 的半圓，其中 AB 是直徑，C 是 AB 上的一點。D 是圓周上的一點使得 $DC \perp AB$。若 $AC = 12$，求 CD，答案準確至最接近整數。（5 分）



The figure shows a semi-circle with radius 20. AB is the diameter, C is a point on AB and D is a point on the circumference such that $DC \perp AB$. If $AC = 12$, find CD correct to the nearest integer. (5 marks)

12. 已知某五位平方數只有數字 2 和 5，求該平方數的最後四位數字。（6 分）

It is known that a five-digit square number contains the digits 2 and 5 only. Find the last four digits of the square number. (6 marks)

13. 若方程 $x^{\log x} = 2010$ 的根之積爲 p，求 $10p^2$ 的值。（6 分）

If the product of roots of the equation $x^{\log x} = 2010$ is p, find the value of $10p^2$. (6 marks)

14. 某個三角柱體每條邊的長度皆相同。如果把它 5 個面的中心連起，我們可以得到一個有 5 個頂點的立體。三角柱體的體積是該立體體積的多少倍？（6 分）

A triangular prism has all edges of the same length. By joining the centre of each of the 5 faces of the prism, we get a solid with 5 vertices. How many times the volume of the solid is the volume of the prism? (6 marks)

15. 已知 n 是兩位正整數。當 n^2 除以 10 時，商是奇數。求 n 的所有可能值之和。（6 分）

Given n is a two-digit positive integer. When n^2 is divided by 10, the quotient is an odd number. Find the sum of all possible values of n. (6 marks)

16. 在一張咭紙上，式子 $x^2-\square x+\square=0$ 被印上 500 次，即共有 1000 個空格。現有 1000 張貼紙，分別印上 1、2、…、1000，並容許在每個空格貼上一張貼紙，從而得到 500 條二次方程。在所得的方程中，有重根的最多有幾條？（6 分）

On a cardboard the expression $x^2-\square x+\square=0$ is printed 500 times, so that there are altogether 1000 blanks. One is given 1000 stickers, with the numbers 1, 2, …, 1000 printed on them, and is allowed to put one sticker onto each blank, resulting in 500 quadratic equations. At most how many of the resulting equations can have a double root? (6 marks)

17. 某國家舉行總統選舉。點票的地方有一個大螢幕，顯示已點選票中得票最多的候選人的照片，每點一票後結果會立即更新。若票數相同（例如開始時各人均得 0 票），則顯示最高票的候選人中姓氏筆劃最少的一人的照片。已知四名分別姓丁、方、林和曾的候選人最終分別得到 50、100、150 和 500 票，則螢幕顯示的照片最多會被切換幾次？（6 分）

A presidential election is held in a country. At the venue where the ballots are counted, there is a screen showing the photo of the candidate on the lead, and is updated after each ballot is being counted. In case of a tie (e.g. all candidates have 0 counted votes at the beginning), then the photo of the candidate among the leaders whose surname ranks first alphabetically will be shown. If the four candidates, with surnames Ding, Fong, Lam and Tsang, got 50, 100, 150 and 500 votes respectively, for at most how many times will the photo shown be changed? (6 marks)

18. 某國家推出自訂車牌計劃，市民可自選一個由最少 2 個和最多 8 個大寫英文字母及/或數字組成的車牌，但不可使用字母 I、O 和 Q。例如：「PC1MC09」和「1L0VEMAT」都是可以的，但「A」、「QUEEN」和「MATH1SFUN」則不能使用。若可使用的車牌共有 n 個，求 n 除以 32 時的餘數。（6 分）

In a country there is a personalised car plate scheme. One can choose a personalised car plate which consists of at least 2 but at most 8 upper case English letters and/or digits, while the letters I, O and Q are not allowed. For instance, 'PC1MC09' and '1L0VEMAT' are allowed, while 'A', 'QUEEN' and 'MATH1SFUN' are not allowed. If there are a total of n allowable car plates, what is the remainder when n is divided by 32? (6 marks)

19. 在所示的乘式中，每個字母代表一個由 0 至 9 的不同數字。求被乘數 ABC 所代表的三位數。 （7 分）

In the multiplication shown, each letter represents a different digit from 0 to 9. Find the three-digit number represented by the multiplicand ABC. (7 marks)

$$\begin{array}{rr} & A\ B\ C \\ \times & \quad B\ C \\ \hline A\ A\ B\ B\ A & \end{array}$$

20. 某數學競賽共設 20 題，每題的佔分皆是 2 至 10 之間的整數（包括 2 和 10），總分爲 100。每題答案正確可得該題所有分數，否則該題得 0 分。大玲答對了 a 題，小玲答對了 z 題，而大玲的得分比小玲高。求 $100+a-z$ 的最小可能值。 （7 分）

There are 20 problems in a mathematical competition, each carrying an integral score between 2 and 10 (inclusive). The total score is 100. Full marks to a problem are given for a correct answer, and no mark is given otherwise. Amy answered a questions correctly and Zoe answered z questions correctly. If the score of Amy is higher than that of Zoe, find the smallest possible value of $100+a-z$. (7 marks)

全卷完

END OF PAPER