第九屆培正數學邀請賽

9th Pui Ching Invitational Mathematics Competition

初賽（中三組）

Heat Event (Secondary 3)

時限：1 小時 15 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

(a) 本卷共設 20 題，總分爲 100 分。

There are 20 questions in this paper and the total score is 100.

(b) 除特別指明外，本卷內的所有數均爲十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

(c) 所有答案皆是 0 至 9999 之間的整數（包括 0 和 9999）。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

All answers are integers between 0 and 9999 (including 0 and 9999). Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

(d) 不得使用計算機。

The use of calculators is not allowed.

(e) 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

1. 當 20100123 除以 11 時，餘數是多少？ （3 分）

What is the remainder when 20100123 is divided by 11? (3 marks)

2. 某數加上自己的個位數字後，結果是 2010。求某數的最小可能值。 （3 分）

When a number is added to its unit digit, the result is 2010. Find the smallest possible value of the number. (3 marks)

3. 某圖形的旋轉對稱次數是 8。若把它繞著旋轉對稱中心順時針旋轉 2010°，過程中會跟原圖形重疊多少次（不計開始時的重疊）？ （3 分）

The order of rotational symmetry of a figure is 8. If the figure is rotated about the centre of rotational symmetry clockwise through 2010°, how many times will the original object be overlapped (not counting the one at the beginning)? (3 marks)

4. 若一個邊長 12、18、24 的三角形和另一個邊長 32、m、n 的三角形相似，則 m 有多少個不同的可能值？ （4 分）

If a triangle with side lengths 12, 18, 24 is similar to another triangle with side lengths 32, m, n, how many different possible values of m are there? (4 marks)

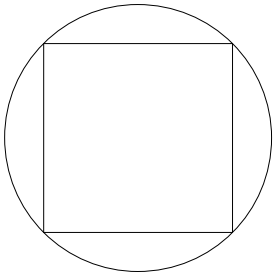
5. 求 20100123^2 的首四位數字。 （4 分）

Find the first four digits of 20100123^2. (4 marks)

6. 有多少個四位正整數的首兩位數字和末兩位數字均是一個兩位的平方數？ （4 分）

How many four-digit positive integers are there such that the first two digits and the last two digits both form a two-digit square number? (4 marks)

7. 如圖所示，一個正方形內接於一個圓形。若圓形的面積是正方形的面積的 k 倍，求最接近 $100k$ 的整數。 （4 分）



In the figure, a square is inscribed in a circle. If the area of the circle is k times the area of the square, find the integer closest to $100k$. (4 marks)

8. 若 p 是大於 2010 的質數，則 p^2 被 8 除時的餘數是多少？（5 分）

If p is a prime number greater than 2010, what is the remainder when p^2 is divided by 8? (5 marks)

9. 現有 50 個互不相等的正整數，它們之和是 2010。若當中最小的一個整數是 n，求 n 的最大可能值。（5 分）

There are 50 pairwise different positive integers with sum 2010. If the smallest integer is n, find the greatest possible value of n. (5 marks)

10. 有多少種方法把整數 1 至 9 分成三組，使得每組皆有三個數，且每組的三個數之和皆相同？（5 分）

In how many different ways can the integers 1 to 9 be divided into three groups such that each group consists of three numbers and the sums of the three numbers in each of the three groups are the same? (5 marks)

11. 當 2010 被一個兩位正整數 n 除時，餘數的最大可能值是甚麼？（5 分）

When 2010 is divided by a two-digit positive integer n, what is the greatest possible value of the remainder? (5 marks)

12. 某班女生的數目是男生的兩倍。一次測驗後，老師決定給每位女同學多加 30 分。這樣，全班的平均分會提高多少分？（5 分）

In a class, there are twice as many girls as boys. After a test, the teacher decided to add 30 marks to each girl. By how many marks will the class average increase as a result? (5 marks)

13. 若每個字母代表一個 0 至 9 的不同數字，並已知五位數 PCIMC 可被 5 整除，則 PCIMC 有多少個不同的可能值？（6 分）

If each letter represents a different digit from 0 to 9 and it is known that the five-digit number PCIMC is divisible by 5, how many different possible values of PCIMC are there? (6 marks)

14. 觀察以下規律。若依規律一直寫到第 10 行，則所有出現在第 10 行中的整數之和是多少？（重複出現的整數亦需計算在內。） （6 分）

Observe the following pattern. If the pattern is carried on until the 10th row, what is the sum of all the integers which occur in the 10th row? (Repeatedly occurring integers have to be included into the sum.) (6 marks)

$$1\times(1+3)=4$$
$$1\times2\times(1+3+6)=4\times5$$
$$1\times2\times3\times(1+3+6+10)=4\times5\times6$$
$$\vdots$$

15. $ABCD$ 是長方形。它的對角線長 10 厘米。若將長方形沿對角線 AC 摺疊，摺疊後 B 和 D 的距離是 4 厘米。求長方形的面積（以平方厘米爲單位），答案準確至最接近整數。 （6 分）

$ABCD$ is a rectangle. Its diagonal is 10 cm long. If the rectangle is folded along AC, the distance between B and D is 4 cm after folding. Find the area of the rectangle (in cm^2) correct to the nearest integer. (6 marks)

16. 已知 A、B、C、D 爲合成數，且 L 是它們的公倍數。若 A、B、C、D 的最小質因數互不相同，求 L 的最小可能值。 （6 分）

Let A, B, C, D be composite numbers and L be their common multiple. If the smallest prime factors of A, B, C, D are pairwise different, find the smallest possible value of L. (6 marks)

17. 設 $[x]$ 代表不超過 x 的最大整數，例如 [1.1] = 1、[6.9] = 6 和 [5] = 5。已知 y 和 z 都是小於 10 的正數，求 $[yz-[y][z]]$ 的最大可能值。 （6 分）

Let $[x]$ denote the greatest integer not exceeding x. For example, [1.1] = 1, [6.9] = 6 and [5] = 5. Given that y and z are positive numbers less than 10, find the greatest possible value of $[yz-[y][z]]$. (6 marks)

18. 如果某天的「年」、「月」、「日」中的所有數字皆不超過 3，則那天稱爲「好日子」，例如：2010 年 1 月 23 日是「好日子」。那麼，在 21 世紀（2001 年 1 月 1 日至 2100 年 12 月 31 日）中，「好日子」共有多少天？ （6 分）

If all digits in the 'year', 'month' and 'day' of a date do not exceed 3, we say that the day is 'good'. For instance, 2010/1/23 is a 'good day'. How many 'good days' are there in the 21st century (from 2001/1/1 to 2100/12/31)? (6 marks)

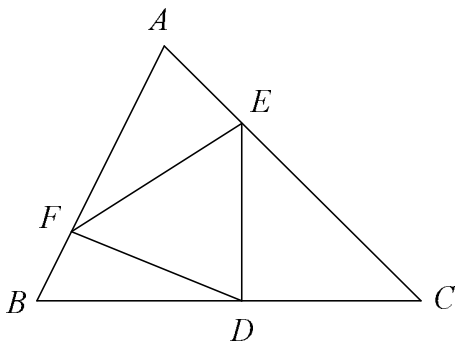
19. 在所示的乘式中，每個字母代表一個由 0 至 9 的不同數字。求被乘數 ABCC 所代表的四位數。 （7 分）

In the multiplication shown, each letter represents a different digit from 0 to 9. Find the four-digit number represented by the multiplicand ABCC. (7 marks)

$$\begin{array}{rr} & A\ B\ C\ C \\ \times & \quad A\ B \\ \hline A\ C\ C\ B\ C\ B & \end{array}$$

20. 圖中，*AFB*、*BDC* 和 *CEA* 是直線，且 *DE* ⊥ *BC*、*EF* ⊥ *CA*、*FD* ⊥ *AB*。若 *AF* = 25、*FB* = 18、*BD* = 30 且 *EF* = *ED*，求 *CE* 的長度，答案準確至最接近整數。 （7 分）

In the figure, *AFB*, *BDC* and *CEA* are straight lines and *DE* ⊥ *BC*, *EF* ⊥ *CA*, *FD* ⊥ *AB*. If *AF* = 25, *FB* = 18, *BD* = 30 and *EF* = *ED*, find the length of *CE* correct to the nearest integer. (7 marks)



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