第九屆培正數學邀請賽

9th Pui Ching Invitational Mathematics Competition

初賽（中二組）

Heat Event (Secondary 2)

時限：1 小時 15 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

(a) 本卷共設 20 題，總分爲 100 分。

There are 20 questions in this paper and the total score is 100.

(b) 除特別指明外，本卷內的所有數均爲十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

(c) 所有答案皆是 0 至 9999 之間的整數（包括 0 和 9999）。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

All answers are integers between 0 and 9999 (including 0 and 9999). Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

(d) 不得使用計算機。

The use of calculators is not allowed.

(e) 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

1. 當 20100123 除以 30 時，餘數是多少？ （3 分）

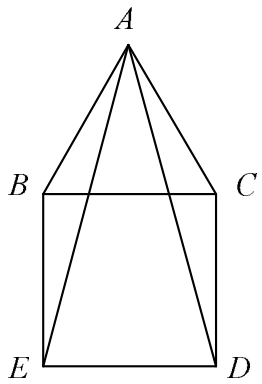
What is the remainder when 20100123 is divided by 30? (3 marks)

2. 小嘉寫下了所有小於 2010 的質數。在寫下的數中，有多少個不同的個位數字？ （3 分）

Chris wrote down all prime numbers less than 2010. Among the numbers written, how many different unit digits are there? (3 marks)

3. 圖中，ABC 是等邊三角形而 $BCDE$ 是正方形。若 $\angle EAD = x°$，求 x。 （3 分）

In the figure, ΔABC is equilateral while $BCDE$ is a square. If $\angle EAD = x°$, find x. (3 marks)



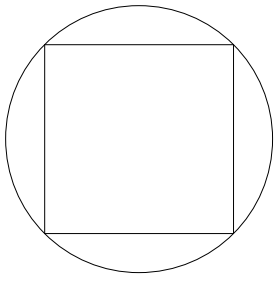
4. 觀察以下規律。若依規律一直寫到第 10 行，則第 10 行等號左方的最後一個整數是甚麼？ （3 分）

Observe the following pattern. If the pattern is carried on until the 10th row, what is the last integer on the left side of the equality in the 10th row? (3 marks)

$$1\times(1+3)=4$$
$$1\times2\times(1+3+6)=4\times5$$
$$1\times2\times3\times(1+3+6+10)=4\times5\times6$$
$$\vdots$$

5. 如圖所示，一個正方形內接於一個圓形。若圓形的面積是正方形的面積的 k 倍，求最接近 $100k$ 的整數。 （4 分）

In the figure, a square is inscribed in a circle. If the area of the circle is k times the area of the square, find the integer closest to $100k$. (4 marks)

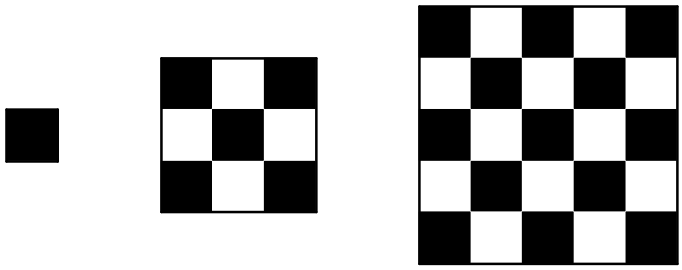


6. 求 20100123^2 的首四位數字。 （4 分）

Find the first four digits of 20100123^2. (4 marks)

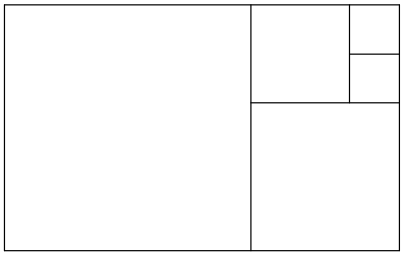
7. 如圖所示，若依首三幅圖的規律一直畫下去，則第 10 幅圖中有多少個白色的小正方形？ （4 分）

If the figure below is continued to be drawn based on the pattern of the first 3 pictures, how many little white squares will there be in the 10th picture? (4 marks)



8. 圖中，一個長方形被分成 5 個正方形。若當中最大的一個正方形的面積是 3600，求長方形的面積。 （4 分）

In the figure, a rectangle is divided into 5 squares. If the largest square has area 3600, find the area of the rectangle. (4 marks)



9. 某數學競賽共設 20 題，首 10 題每題 4 分，後 10 題每題 6 分。每題答案正確可得該題所有分數，否則該題得 0 分。那麼，總得分共有多少個不同的可能性？ （5 分）

In a mathematical competition there are 20 questions; the first 10 carries 4 marks each while the last 10 carries 6 marks each. Full marks to a problem are given for a correct answer, and no mark is given otherwise. How many different possible total scores are there? (5 marks)

10. 當 2010 被一個兩位正整數 n 除時，餘數是 10。問 n 有多少個不同的可能值？ （5 分）

When 2010 is divided by a two-digit positive integer n, the remainder is 10. How many different possible values of n are there? (5 marks)

11. 在 2 時 34 分，時鐘上時針和分針所成的鈍角是 $x°$；在 4 時 32 分，時鐘上時針和分針所成的銳角是 $y°$。求 $x-y$。（5 分）

At 2:34, the obtuse angle formed by the hour hand and the minute hand of a clock is $x°$; at 4:32, the acute angle formed by the hour hand and the minute hand of a clock is $y°$. Find $x-y$. (5 marks)

12. 設 $[x]$ 代表不超過 x 的最大整數，例如 $[1.1] = 1$、$[6.9] = 6$ 和 $[5] = 5$。已知 y 和 z 都是小於 10 的正數，求 $[yz-[y][z]]$ 的最大可能值。（6 分）

Let $[x]$ denote the greatest integer not exceeding x. For example, $[1.1] = 1$, $[6.9] = 6$ and $[5] = 5$. Given that y and z are positive numbers less than 10, find the greatest possible value of $[yz-[y][z]]$. (6 marks)

13. 天樂有兩張全等的三角形紙。他用三種不同的方法將它們拼成平行四邊形。這三個平行四邊形的周界分別是 24、33 和 35。求三角形紙的周界。（6 分）

Tony had two congruent sheets of paper in triangular shape. He built a parallelogram with them by three different methods. It is known that the perimeters of the three parallelograms are 24, 33 and 35. Find the perimeter of the triangular piece of paper. (6 marks)

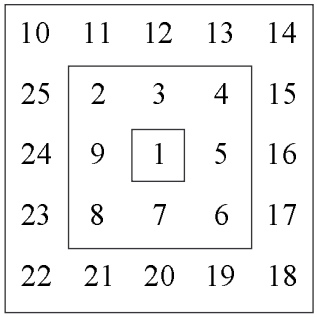
14. $ABCD$ 是長方形。它的對角線長 10 厘米。若將長方形沿對角線 AC 摺疊，摺疊後 B 和 D 的距離是 4 厘米。求長方形的面積（以平方厘米爲單位），答案準確至最接近整數。（6 分）

$ABCD$ is a rectangle. Its diagonal is 10 cm long. If the rectangle is folded along AC, the distance between B and D is 4 cm after folding. Find the area of the rectangle (in cm^2) correct to the nearest integer. (6 marks)

15. 一家商店舉行特價酬賓活動，所有貨品均以標價的六折出售。其中一件貨品標價 n 元，折扣後售價 m 元，其中 m 和 n 都是三位數，而且兩者由相同的數字組成（只是次序不同）。求 n。（6 分）

A shop is on sale and all goods are sold at 40% off the marked price. One article is marked for n dollars and the discounted price is m dollars, where m, n are both three-digit numbers made up of the same digits (with different orders). Find n. (6 marks)

16. 圖中所示爲一個整數的陣列，其組成如下：先在中間的正方形寫 1，然後畫出第二重正方形，再從左上角開始以順時針方向寫上 2 至 9。之後畫出第三重正方形，再以相同方法從左上角開始以順時針方向寫上 10 至 25。若依此方法一直寫下去，則 2010 右邊的整數是甚麼？（6 分）



The figure shows an array of integers formed as follows. First, the number 1 is written in the central square. Then a second level of square is drawn and the numbers 2 to 9 are written in clockwise order starting from the top left corner. Similarly the third level of square is then drawn and the numbers 10 to 25 are written in clockwise order starting from the top left corner. If this pattern continues, what is the number on the right of 2010? (6 marks)

17. 如果某天的「年」、「月」、「日」中的所有數字皆不超過 3，則那天稱爲「好日子」，例如：2010 年 1 月 23 日是「好日子」。那麼，在 21 世紀（2001 年 1 月 1 日至 2100 年 12 月 31 日）中，「好日子」共有多少天？（6 分）

If all digits in the 'year', 'month' and 'day' of a date do not exceed 3, we say that the day is 'good'. For instance, 2010/1/23 is a 'good day'. How many 'good days' are there in the 21st century (from 2001/1/1 to 2100/12/31)? (6 marks)

18. 一部特別的計算機上有一個紅色按鈕。當按下這個紅色按鈕時，螢幕上的數會被加上它的個位數字。例如：如果螢幕上的數是 27，則按下紅色按鈕後該數會變成 $27+7=34$，再按下紅色按鈕該數會變成 $34+4=38$，如此類推。現有一群學生，他們的編號分別是 1、11、21、…、111，且每人手上均有一部這樣的計算機。他們先把自己的編號輸入計算機，然後每人按下紅色按鈕 10 次，那麼各人所得的結果之和是多少？（7 分）

A special calculator contains a red key. When the red key is pressed, the number on the screen will be added by its unit digit. For instance, if the screen shows 27 and the red key is pressed, the number on screen becomes $27+7=34$; if the red key is pressed again the number on screen becomes $34+4=38$, and so on. A group of students, numbered 1, 11, 21, …, 111, each with such a calculator, first input their numbers into the calculator and each pressed the red key 10 times. What is the sum of their results? (7 marks)

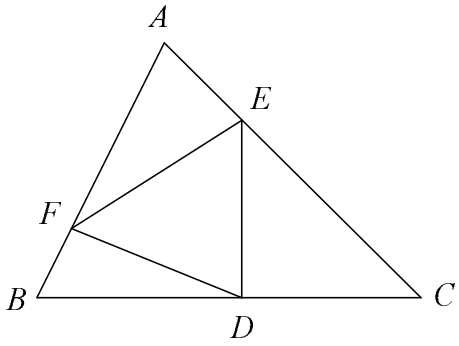
19. 在所示的乘式中，每個字母代表一個由 0 至 9 的不同數字。求被乘數 ABCC 所代表的四位數。 （7 分）

In the multiplication shown, each letter represents a different digit from 0 to 9. Find the four-digit number represented by the multiplicand ABCC. (7 marks)

$$\begin{array}{rr} & A\ B\ C\ C \\ \times & \quad A\ B \\ \hline A\ C\ C\ B\ C\ B & \end{array}$$

20. 圖中，*AFB*、*BDC* 和 *CEA* 是直線，且 *DE* ⊥ *BC*、*EF* ⊥ *CA*、*FD* ⊥ *AB*。若 *AF* = 25、*FB* = 18、*BD* = 30 且 *EF* = *ED*，求 *CE* 的長度，答案準確至最接近整數。 （7 分）

In the figure, *AFB*, *BDC* and *CEA* are straight lines and *DE* ⊥ *BC*, *EF* ⊥ *CA*, *FD* ⊥ *AB*. If *AF* = 25, *FB* = 18, *BD* = 30 and *EF* = *ED*, find the length of *CE* correct to the nearest integer. (7 marks)



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