第九屆培正數學邀請賽

9th Pui Ching Invitational Mathematics Competition

初賽（中一組）

Heat Event (Secondary 1)

時限：1 小時 15 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

(a) 本卷共設 20 題，總分爲 100 分。

There are 20 questions in this paper and the total score is 100.

(b) 除特別指明外，本卷內的所有數均爲十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

(c) 所有答案皆是 0 至 9999 之間的整數（包括 0 和 9999）。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

All answers are integers between 0 and 9999 (including 0 and 9999). Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

(d) 不得使用計算機。

The use of calculators is not allowed.

(e) 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

1. 當 20100123 除以 20 時，餘數是多少？ （3 分）

What is the remainder when 20100123 is divided by 20? (3 marks)

2. 若把一個正方形切成兩個相同大小的長方形，則其總周長會增加 $n\%$。求 n。 （3 分）

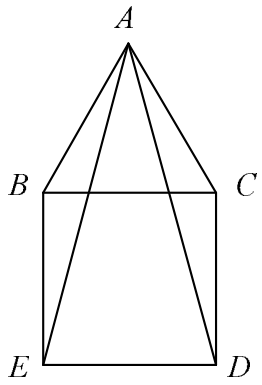
When a square is cut into two equal rectangles, the total perimeter is increased by $n\%$. Find n. (3 marks)

3. 求 123456×999998 的最後四位數字。 （3 分）

Find the last four digits of 123456×999998. (3 marks)

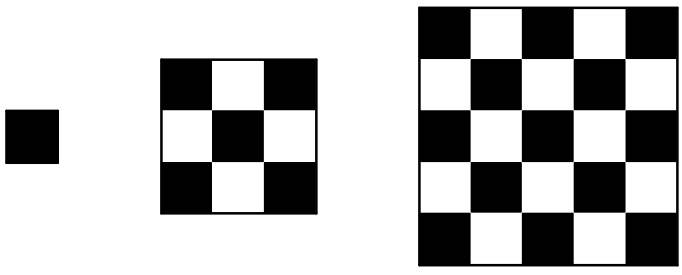
4. 圖中，ABC 是等邊三角形而 $BCDE$ 是正方形。若 $\angle EAD = x°$，求 x。 （3 分）

In the figure, ΔABC is equilateral while $BCDE$ is a square. If $\angle EAD = x°$, find x. (3 marks)



5. 如下圖依首三幅圖的規律一直畫下去直至第 100 幅，則這 100 幅圖中黑色小正方形和白色小正方形的總數會相差多少？ （3 分）

If the figure below is continued to be drawn based on the pattern of the first 3 pictures until the 100th picture, what is the difference between the total number of little black squares and the total number of little white squares in the 100 pictures? (3 marks)



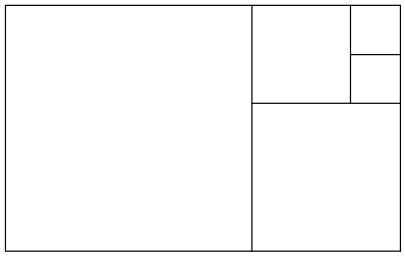
6. 在 1 時 23 分，時鐘上時針和分針所成的角是 $x°$；在 10 時 n 分（其中 n 是整數），時鐘上時針和分針所成的角也是 $x°$。求 n。 （4 分）

At 1:23, the angle formed by the hour hand and the minute hand of a clock is $x°$; at n minutes past ten (where n is an integer), the angle formed by the hour hand and the minute hand of a clock is also $x°$. Find n. (4 marks)

7. 兩個全等的錐體的底都是正六邊形。若把兩個錐體的底黏合起來使它們重合，可得到一個新的立體。這個立體有多少條邊？ （4 分）

The bases of two congruent pyramids are both regular hexagons. If the bases are stuck together so that they coincide, a new solid is formed. How many edges are there in the resulting solid? (4 marks)

8. 圖中，一個長方形被分成 5 個正方形。若當中最大的一個正方形的面積是 3600，求長方形的面積。 （4 分）



In the figure, a rectangle is divided into 5 squares. If the largest square has area 3600, find the area of the rectangle. (4 marks)

9. 若把 $\frac{1}{\sqrt{20100123}}$ 寫成小數，則小數點後首四位數字是甚麼？ （5 分）

When $\frac{1}{\sqrt{20100123}}$ is expressed as a decimal, what are the first four digits after the decimal point? (5 marks)

10. 當 2010 被一個兩位正整數 n 除時，餘數是 1。問 n 有多少個不同的可能值？ （5 分）

When 2010 is divided by a two-digit positive integer n, the remainder is 1. How many different possible values of n are there? (5 marks)

11. 在首 2010 個正整數中，最少要抽出多少個，才可保證當中必定有兩個的最大公因數是 1？ （5 分）

What is the smallest number of integers that must be chosen from the first 2010 positive integers to ensure that there must exist two chosen numbers whose H.C.F. is 1? (5 marks)

12. 天樂有兩張全等的三角形紙。他用三種不同的方法將它們拼成平行四邊形。這三個平行四邊形的周界分別是 24、33 和 35。求三角形紙的周界。（6 分）

Tony had two congruent sheets of paper in triangular shape. He built a parallelogram with them by three different methods. It is known that the perimeters of the three parallelograms are 24, 33 and 35. Find the perimeter of the triangular piece of paper. (6 marks)

13. 一家商店舉行特價酬賓活動，所有貨品均以標價的六折出售。其中一件貨品標價 n 元，折扣後售價 m 元，其中 m 和 n 都是三位數，而且兩者由相同的數字組成（只是次序不同）。求 n。（6 分）

A shop is on sale and all goods are sold at 40% off the marked price. One article is marked for n dollars and the discounted price is m dollars, where m, n are both three-digit numbers made up of the same digits (with different orders). Find n. (6 marks)

14. 已知 a、b、c 是正整數，其中 $a>b>c$ 且 $(a-b)^2+(a-c)^2+(b-c)^2=74$。求 $(a-b)(a-c)(b-c)$ 的值。（6 分）

Given a, b, c are positive integers such that $a>b>c$ and $(a-b)^2+(a-c)^2+(b-c)^2=74$. Find the value of $(a-b)(a-c)(b-c)$. (6 marks)

15. 圖中所示爲一個整數的陣列，其組成如下：先在中間的正方形寫 1，然後畫出第二重正方形，再從左上角開始以順時針方向寫上 2 至 9。之後畫出第三重正方形，再以相同方法從左上角開始以順時針方向寫上 10 至 25。若依此方法一直寫下去，則由整數 1 開始往右數的第 10 個數（不包括 1）是甚麼？（6 分）

10	11	12	13	14
25	2	3	4	15
24	9	1	5	16
23	8	7	6	17
22	21	20	19	18

The figure shows an array of integers formed as follows. First, the number 1 is written in the central square. Then a second level of square is drawn and the numbers 2 to 9 are written in clockwise order starting from the top left corner. Similarly the third level of square is then drawn and the numbers 10 to 25 are written in clockwise order starting from the top left corner. If this pattern continues, what is the 10th number on the right of the number 1 (not counting the number 1)? (6 marks)

16. 一部特別的計算機上有一個紅色按鈕。當按下這個紅色按鈕時，螢幕上的數會被加上它的個位數字。例如：如果螢幕上的數是 27，則按下紅色按鈕後該數會變成 $27+7=34$，再按下紅色按鈕該數會變成 $34+4=38$，如此類推。現於計算機輸入一個正整數，再按下紅色按鈕若干次後，螢幕上的數變成 1234。那麼開始時輸入的整數的最小可能值是甚麼？（6 分）

A special calculator contains a red key. When the red key is pressed, the number on the screen will be added by its unit digit. For instance, if the screen shows 27 and the red key is pressed, the number on screen becomes $27+7=34$; if the red key is pressed again the number on screen becomes $34+4=38$, and so on. A positive integer is now input to the calculator. The red button is then pressed some number of times, and the integer on the screen becomes 1234. What is the smallest possible value of the integer input? (6 marks)

17. 某國家有五種硬幣，面值分別是 1 元、5 元、10 元、25 元和 d 元。已知任意不超過 100 元的整數金額均可用不超過 7 個硬幣組成，求 d 的最大可能值。（7 分）

There are five types of coins in a country, with denominations 1 dollar, 5 dollars, 10 dollars, 25 dollars and d dollars respectively. Given that any integral amount not exceeding 100 dollars can be formed using not more than 7 coins, find the greatest possible value of d. (7 marks)

18. 小克和小勤在路的兩端相向而行。兩人同時出發，而且都保持均速。他們在出發後 15 分鐘相遇。假如小克的速度是原來的兩倍而小勤的速度不變，他們在出發後 12 分鐘便相遇。若小克的速度是原來的三倍而小勤的速度不變，他們在出發後多少分鐘相遇？（7 分）

Ben and Carl were walking towards each other from the two ends of a road. They started the journey at the same time and walked at uniform speed. They met each other 15 minutes after they had started. If Ben walked at twice his original speed with Carl's speed unchanged, they could meet each other 12 minutes after they had started. If Ben walked at three times his original speed with Carl's speed unchanged, how many minutes would it take for them to meet after they had started? (7 marks)

19. 在首 2010 個正整數中，最少有兩個連續數字相同的（例如：688、1199、1333）有多少個？（7 分）

How many of the first 2010 positive integers have at least two consecutive digits equal (e.g. 688, 1199, 1333)? (7 marks)

20. 在一個遊戲中，有從左至右編號爲 1 至 2010 的 2010 個座位。開始時，每個奇數號碼的座位上均有一名參賽者，其中座位編號的個位數字爲 9 的稱爲「右派」參賽者，其他參賽者則稱爲「左派」。在每個回合中，每位左派參賽者均向左移一個座位，每位右派參賽者均向右移一個座位。如果左/右邊已經沒有座位可供移動，則有關參賽者將離開遊戲。如果某回合中兩名參賽者（左派和右派各一）移至同一座位，則從下一回合開始，當中的左派將變成右派，右派則變成左派。遊戲一直繼續，直至只剩下一名參賽者爲止。遊戲共需進行多少個回合？ （7 分）

In a game, there are 2010 seats, numbered 1 to 2010 from left to right. Initially, there is a player on every odd-numbered seat. Those sitting on a seat with unit digit 9 are 'rightists'; the other players are 'leftists'. In each round, each leftist moves to the seat on his left while each rightist moves to the seat on his right. If no seat is available on his left/right, the player leaves the game. If two players (a leftist and a rightist) move to the same seat in a certain round, the leftist becomes a rightist and the rightist becomes a leftist starting from the next round. The game continues until only one player remains. For how many rounds will the game last? (7 marks)

全卷完

END OF PAPER