第十八屆培正數學邀請賽（**2019** 年）

18th Pui Ching Invitational Mathematics Competition (2019)

初賽（中四組）

Heat Event (Secondary 4)

時限：**1** 小時 **15** 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

(a) 本卷共設 20 題，總分為 100 分。

There are 20 questions in this paper and the total score is 100.

(b) 除特別指明外，本卷內的所有數均為十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

(c) 作答時，每題的答案均須以 0 至 9999 之間的整數表示。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

Each answer must be given in the form of an integer between 0 and 9999. Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

(d) 不得使用計算機。

The use of calculators is not allowed.

(e) 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

注意：每題的答案均須以 0 至 9999 之間的整數表示，如有需要應以上述範圍內最接近正確答案的整數回答。如有兩個這樣的整數與正確答案同樣接近，則以「四捨五入」的原則取較大的整數。請細閱答題紙上的指示。

Note: Each answer must be given in the form of an integer between 0 and 9999. Where necessary, the answer should be rounded off to the nearest integer in the above range. Read the instructions on the answer sheet in detail.

1. 有多少個不超過 10 的正整數 n 滿足 $-6n \leq -42$？（3 分）

   How many positive integers n not exceeding 10 satisfy $-6n \leq -42$? (3 marks)

2. 設 a 為實數。若方程 $(x-3)(2x-12)(3x-a)=0$ 有重根，求 a 的所有可能值之和。（3 分）

   Let a be a real number. If the equation $(x-3)(2x-12)(3x-a)=0$ has repeated roots, find the sum of all possible values of a. (3 marks)

3. 現有一個實數等比數列，其中公比不是 1。那麼，最多可以有多少個連續的質數項？（3 分）

   There is a geometric sequence of real numbers. The common ratio is not 1. At most how many consecutive prime number terms are there? (3 marks)

4. 一個長方體其中三邊的邊長比為 $1:2:3$。若長方體的體積為 384，求它最長邊的長度。（3 分）

   The lengths of three sides of a cuboid are in the ratio $1:2:3$. If the volume of the cuboid is 384, find the length of its longest side. (3 marks)

5. 在一個 9×9 棋盤的左上角方格中有一枚棋子，每步我們可把棋子移動至一個有相鄰邊的方格內，且同一方格只可經過一次（左上角方格除外）。經過 n 步後，棋子首次返回起始位置。求 n 的最大可能值。（4 分）

   A chess piece is placed in the top left corner cell of a 9×9 chessboard. In each step we can move the chess piece to an adjacent cell sharing a common side, while each cell (except for the top left corner one) may only be visited once. After n steps, the chess piece returns to the original position for the first time. Find the greatest possible value of n. (4 marks)

6. 在所示的算式中，每個字母代表一個由 0 至 9 的不同數字。求 CD 所代表的兩位數。 （4 分）

In the addition shown, each letter represents a different digit from 0 to 9. Find the two-digit number represented by CD. (4 marks)

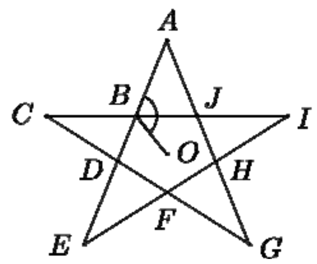
$$\begin{array}{rccc} & & A & B \\ + & & B & A \\ \hline & C & C & D \end{array}$$

7. 有些人在進行一項工作，其中每人獨自工作時的效率都是相同的。不過，每當有額外一人一起工作時，每個人的工作效率均會減低 30%。若希望工作在最短時間內完成，則應該由多少人一起工作？ （4 分）

Some people are working on a job. The individual working efficiency of each person is the same. However, for each additional person working together, the efficiency of each person will be reduced by 30%. What should be the number of people working in order to complete the job in the shortest time? (4 marks)

8. 現有一個星形 *ABCDEFGHIJ*，其中頂點 *A*、*C*、*E*、*G*、*I* 是一個正五邊形的頂點，而頂點 *B*、*D*、*F*、*H*、*J* 則是 *ACEGI* 的對角線的交點。設 *O* 是該星形的中心。若 $\angle OBA = x°$，求 x 的值。 （4 分）

There is a star shape *ABCDEFGHIJ*. The vertices *A*, *C*, *E*, *G*, *I* are the vertices of a regular pentagon, while the vertices *B*, *D*, *F*, *H*, *J* are the intersections of the diagonals of *ACEGI*. Let *O* be the centre of the star shape. If $\angle OBA = x°$, find the value of x. (4 marks)



9. 現有 2019 顆勻稱的骰子，當中每顆骰子的六個面分別寫上 0 至 5 的整數。甲投擲其中一顆骰子並記下擲出的點數，之後乙投擲全部 2019 顆骰子並記下所得點數之和除以 6 的餘數。若二人所記下的數相同的概率以最簡分數表示時為 $\frac{a}{b}$，求 $a+b$ 的值。 （5 分）

There are 2019 fair dice, each with the integers from 0 to 5 written on the six faces. Ann throws one of the dice and records the number obtained, while Ben throws all 2019 dice and records the remainder when the sum of the numbers obtained is divided by 6. If the probability that they record the same number is $\frac{a}{b}$ in lowest form, find the value of $a+b$. (5 marks)

10. 在一個遊戲中，每集齊 5 張咭，其中 3 張編號相同而另外 2 張亦編號相同，則可以換領一份獎品。現有 40 張咭，其中編號為 0、1、⋯、9 的各佔 4 張。那麼，最多可以換領多少份獎品？ （5 分）

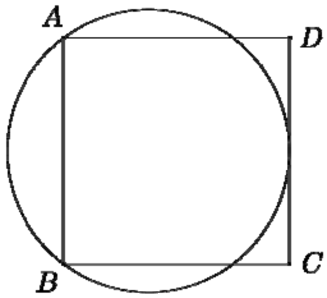
In a game, one can redeem a prize with every collection of 5 cards in which 3 have the same label and the other 2 have the same label. Now there are 40 cards, with 4 cards each of labels 0, 1, … 9. What is the maximum number of prizes that can be redeemed? (5 marks)

11. 黑板上有 n 個正整數，其中首 9 個分別是 1 至 9，之後有若干個是 5，最後的若干個則是 8。若這些正整數的平均值為 $\frac{32}{5}$，求 n 的最小可能值。 （5 分）

There are n positive integers on the blackboard. The first 9 of them are 1 to 9. Then there are some copies of 5 followed by some copies of 8 at the end. If the mean of all numbers on the blackboard is $\frac{32}{5}$, find the smallest possible value of n. (5 marks)

12. 設 $ABCD$ 為邊長為 40 的正方形。一個穿過 A 和 B 的圓形與 CD 相切。若該圓形的面積是 $k\pi$，求 k 的值。 （5 分）

Let $ABCD$ be a square with side length 40. A circle passing through A and B is tangent to CD. If the area of the circle is $k\pi$, find the value of k. (5 marks)



13. 在首 100 個正整數中，有多少個不包含數字「3」而且不是 3 的倍數？ （5 分）

How many of the first 100 positive integers do not contain the digit ‘3’ and are not multiples of 3? (5 marks)

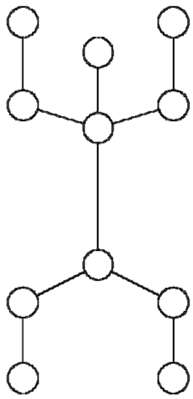
14. 設 a、b、c 為正整數，使得 $a+b+c=1321$。求 $\sqrt{abc}$ 的最大可能值。 （6 分）

Let a, b, c be positive integers such that $a+b+c=1321$. Find the greatest possible value of $\sqrt{abc}$. (6 marks)

15. 某班 30 名學生的班號分別是 1 至 30，他們被分成 n 組。已知任意兩名班號之和為 30 的學生同組，且任意兩名班號之差為 12 的學生亦同組。求 n 的最大可能值。（6 分）

There are 30 students in a class, with class numbers 1 to 30 respectively. They are divided into n groups. It is known that any two students whose sum of class numbers equals 30 belong to the same group, and that any two students whose class numbers differ by 12 also belong to the same group. Find the greatest possible value of n. (6 marks)

16. 現要在圖中的圓形內填上 1 至 11 的整數（不可重複）。如果兩個圓形由直線連接著，則位處較高的圓形內的整數必須大於位處較低的圓形內的整數。那麼，有多少種方法可以完成要求？（7 分）



The integers from 1 to 11 are to be filled into the circles in the figure without repetition. It is required that if two circles are connected by a line, then the integer in the higher circle must be greater than that in the lower circle. How many ways are there to complete this task? (7 marks)

17. 設 P 是 ΔABC 所在的平面上的一點，使得 P 到直線 BC、CA 和 AB 的距離相同。若 $\angle BAC = 72°$ 及 $\angle BPC = x°$，求 x 的所有可能值之和。（7 分）

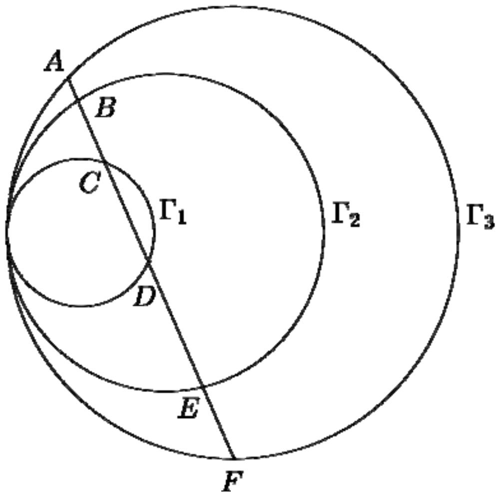
Let P be a point on the plane of ΔABC such that the distances from P to the lines BC, CA and AB are the same. If $\angle BAC = 72°$ and $\angle BPC = x°$, find the sum of all possible values of x. (7 marks)

18. 已知 2^{1068} 的首四位數字等於 5^{1068} 的首四位數字。這四位數字（從左至右）是甚麼？（7 分）

It is given that the first four digits of 2^{1068} are the same as the first four digits of 5^{1068}. What are these four digits (from left to right)? (7 marks)

19. 圓形 Γ_1 、Γ_2 和 Γ_3 在同一點互相內切。一條直線交 Γ_1 於 C 和 D，交 Γ_2 於 B 和 E，並交 Γ_3 於 A 和 F。若 $AB=1$ 、 $BC=2$ 、 $CD=2$ 、 $DE=4$ 及 $EF=x$，求 x 的值。（7 分）

The circles Γ_1 , Γ_2 and Γ_3 are internally tangent to each other at the same point. A straight line meets Γ_1 at points C and D, meets Γ_2 at points B and E, and meets Γ_3 at points A and F. If $AB=1$, $BC=2$, $CD=2$, $DE=4$ and $EF=x$, find the value of x. (7 marks)



20. 哥哥和弟弟在某星期的星期一至星期六都在嘗試解數學題。哥哥在星期一開始時立即解了一道題，弟弟則在星期一正午解了一道題。已知哥哥在任意連續 24 小時中都解了最少一道題，而弟弟則在任意連續 18 小時中解了最少一道題。此外，在星期一、星期三和星期五，首先解出第一道題的是哥哥；在星期二、星期四和星期六，首先解出第一道題的是弟弟。那麼，在這六天中，兄弟二人合共最少解了多少道題？（7 分）

A pair of brothers kept solving mathematical problems from Monday to Saturday in a certain week. The elder brother solved one problem immediately at the beginning of Monday, while the younger brother solved one at noon on Monday. It is known that the elder brother solved at least one problem during any continuous 24-hour period, while the younger brother solved at least one problem during any continuous 18-hour period. Furthermore, it was the elder brother who first solved a problem on Monday, Wednesday and Friday, while it was the younger brother who first solved a problem on Tuesday, Thursday and Saturday. What is the minimum total number of problems solved by the brothers during these six days? (7 marks)

全卷完

END OF PAPER