第十八屆培正數學邀請賽（2019 年）

18th Pui Ching Invitational Mathematics Competition (2019)

初賽（中三組）

Heat Event (Secondary 3)

時限：1 小時 15 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

(a) 本卷共設 20 題，總分為 100 分。

There are 20 questions in this paper and the total score is 100.

(b) 除特別指明外，本卷內的所有數均為十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

(c) 作答時，每題的答案均須以 0 至 9999 之間的整數表示。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

Each answer must be given in the form of an integer between 0 and 9999. Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

(d) 不得使用計算機。

The use of calculators is not allowed.

(e) 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

1. 現有 5 張編號為 1 至 5 的紅色咭片和 5 張編號為 1 至 5 的藍色咭片。若把這 10 張咭片排成一行，最多有多少對相鄰咭片的編號是連續整數（一張咭片可屬於兩對相鄰咭片）？ （3分）

   There are 5 red cards numbered 1 to 5 and another 5 blue cards numbered 1 to 5. If we arrange the 10 cards in a row, at most how many pairs of adjacent cards may have numbers that are consecutive integers (a card can belong to two pairs of adjacent cards)? (3 marks)

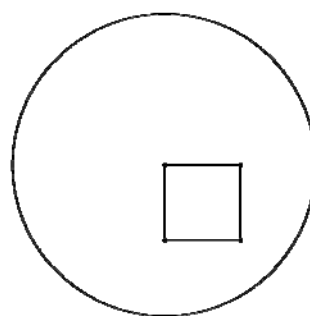2. 一個數列的首三項分別是 2、−1 和 1。若數列每三項重複，求數列首 99 項之和。 （3分）

   The first three terms of a sequence are 2, −1 and 1 respectively. If the sequence repeats every three terms, find the sum of the first 99 terms of the sequence. (3 marks)

3. 現有三個互不相同的正整數，當中任意兩個的最大公因數均不是 1。求這三個整數之和的最小可能值。 （3分）

   There are three pairwise distinct positive integers. If the H.C.F. of any two of them is not equal to 1, find the smallest possible sum of these three integers. (3 marks)

4. 一個半徑為 r 的圓形的圓心位於一個邊長為 4 的正方形的其中一個頂點。求 r 的最小可能值，使得正方形內的任意一點均位於圓形內。 （3分）

   A circle with radius r has its centre lying on one of the vertices of a square with side length 4. Find the smallest possible value of r such that any point inside the square also lies inside the circle. (3 marks)

5. 現有兩個全等的正四面體，並從每個四面體選一個面然後互相黏合，從而得到一個新的立體。那麼，新的立體有多少條棱？　　　　　　　　　　（3分）

Two identical regular tetrahedrons are combined to form a new solid by gluing one face from each tetrahedron to each other. How many edges does the resulting solid have? (3 marks)

6. 我們把整數 1、3、5、7 和 9 填入算式「$(\square \times \square - \square) \times \square - \square$」的方格中，其中每個整數只可使用一次。求所得結果的絕對值的最小可能值。　　（4分）

We fill in the integers 1, 3, 5, 7 and 9 into the cells of the expression '$(\square \times \square - \square) \times \square - \square$' such that each integer is used once. Find the smallest possible value of the absolute value of the result obtained. (4 marks)

7. 某級 n 名學生被分成四組，其中六分之一的學生屬於第一組，四分之一屬於第二或第三組，第四組的人數是第二組的 3 倍，且第三組共有 30 人。求 n 的值。　　　　　　　　　　　　　　　　　　　　　　　　　　　（4分）

A form of n students are divided into four groups. One-sixth of them belong to the first group, one-fourth of them belong to the second or the third group, the number of students in the fourth group is 3 times that of the second group, and there are 30 students in the third group. Find the value of n. (4 marks)

8. 某國家只有 10 元和 25 元面值的硬幣。現有 n 個盒子，每個盒子中都裝有這個國家的硬幣，而且總值均是 750 元。再者，每個盒子中均有最少一個 10 元硬幣和一個 25 元硬幣，而且沒有兩個盒子中的 10 元硬幣數量相同。求 n 的最大可能值。　　　　　　　　　　　　　　　　　　　　　　　（4分）

There are only two types of coins in a country, of denominations $10 and $25 respectively. Now there are n boxes, each containing some coins of this country with total value $750. It is known that each box contains at least one $10 coin and one $25 coin. Furthermore, no two boxes contain the same number of $10 coins. Find the greatest possible value of n. (4 marks)

9. 現有一個正整數數列。這個數列每 4 項重複，而且每 10 項亦重複。若這個數列不是常數數列，求首 2019 項之和的最小可能值。　　　　　　（5分）

There is a sequence of positive integers. This sequence repeats every 4 terms and also repeats every 10 terms. If this sequence is not a constant sequence, find the smallest possible value of the sum of the first 2019 terms. (5 marks)

10. 設 a 和 b 為不小於 1 的實數，且 $a:b=2:3$。求 $\dfrac{9a+6b^2}{3a+b}$ 的最小可能值。　　（5分）

Let a and b be real numbers not less than 1 such that $a:b=2:3$. Find the smallest possible value of $\dfrac{9a+6b^2}{3a+b}$.　(5 marks)

11. 在所示的算式中，每個字母代表一個由 0 至 9 的不同數字。求 ABCD 所代表的四位數。

In the addition shown, each letter represents a different digit from 0 to 9. Find the four-digit number represented by ABCD.

$$
\begin{array}{r}
\text{A B A} \\
+\ \ \text{B B A} \\
\hline
\text{C C D B}
\end{array}
$$

（5分）

(5 marks)

12. 現有一個 8 行 3 列的棋盤。開始時，第一行中間的方格（即圖中填滿色的方格）有一枚棋子，每一步我們可以將棋子移動至一個有公共邊且不曾到過的相鄰方格中。假設棋子經過每行恰好 2 個方格，最後抵達最後一行中間的方格（即圖中畫有斜線的方格）。那麼，棋子行經的路線有多少種不同的可能性？

There is a chessboard with 8 rows and 3 columns. Initially, there is a chess piece in the middle cell of the first row (i.e. the one filled with colour in the figure). In each step, we can move the chess piece to an adjacent cell sharing a common side which has not been visited before. Suppose the chess piece visits exactly 2 cells in each row and eventually reaches the cell in the middle of the last row (i.e. the cell shaded by slashes). How many possibilities are there for the route taken by the chess piece?

（5分）
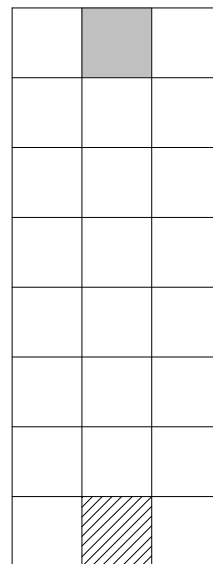
(5 marks)

13. 在一個遊戲開始時，8 人坐在圍成一圈的 8 張椅子上。現要求每人同時移至一張相鄰的椅子，使得每張椅子依然被一個人佔據著。這些人共有多少種不同的方法進行移動？

At the beginning of a game, 8 people are seated around a circle. Now each of them is required to move to an adjacent seat simultaneously, such that each seat remains occupied by one person. In how many different ways can these people move?

（6分）

(6 marks)

14. 有多少個四位數當中任意兩個相鄰數字的奇偶性均不同？ （6分）

    How many four-digit numbers are there such that every two adjacent digits have different parities? (6 marks)

15. 在一個戰艦遊戲中，一艘 1×4 或 4×1 的船隻藏在一個 7×7 的棋盤下，恰好佔據着 4 個方格，但其位置卻是未知的。在每回合中，玩家可以在棋盤的一個方格中投下炸彈，若該炸彈擊中船隻的任何部分便可勝出遊戲。那麼，最少需要多少個回合才可確保勝出？ （6分）
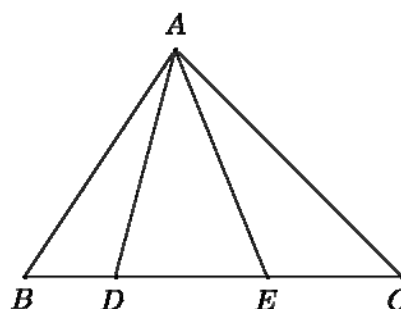
    In a battleship game, a 1×4 or 4×1 ship is hidden in a 7×7 board so that exactly 4 cells of the board are occupied but the position is unknown. In each round, the player can drop a bomb onto a cell of the board, and wins if the bomb hits any part of the ship. At least how many rounds are needed to ensure winning this game? (6 marks)

16. 設 n 為固定的正整數。若方程 $\dfrac{1}{n} = \dfrac{1}{x} + \dfrac{1}{y}$ 剛好共有 2019 組正整數解，則方程 $\dfrac{1}{n} = \dfrac{1}{x} - \dfrac{1}{y}$ 有多少組正整數解？ （7分）

    Let n be a fixed positive integer. If the equation $\dfrac{1}{n} = \dfrac{1}{x} + \dfrac{1}{y}$ has exactly 2019 sets of positive integer solutions, how many sets of positive integer solutions are there to the equation $\dfrac{1}{n} = \dfrac{1}{x} - \dfrac{1}{y}$? (7 marks)

17. 在 ΔABC 中，D 和 E 為 BC 上的點，使得 $\angle DAC = \angle ABC$ 及 $\angle EAB = \angle ACB$ 。若 $BD = 3$、$DE = 6$ 及 $AD = 12$，求 EC 的長度。 （7分）

    In ΔABC, let D and E be points on BC such that $\angle DAC = \angle ABC$ and $\angle EAB = \angle ACB$. If $BD = 3$, $DE = 6$ and $AD = 12$, find the length of EC. (7 marks)
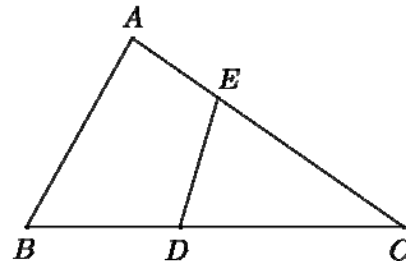
18. 有多少種方法把 P、P、C、C、I、I、M、M、S、S 這 10 個字母排成一行，使得當中連續出現「PCIMC」或「PCMS」其中之一？　　　　　　　（7分）

How many ways are there to permute the 10 letters P, P, C, C, I, I, M, M, S, S in a row such that either 'PCIMC' or 'PCMS' appears consecutively? (7 marks)

19. 在 ΔABC 中，$AB = 7$、$BC = 20$ 及 $CA = 18$。D 和 E 分別是 BC 和 AC 上的點，使得 $\dfrac{BD}{DC} = \dfrac{BA + AE}{EC}$，且 $ABDE$ 的面積等於 ΔCDE 的面積。求 CD 的長度。　　　（7分）

In ΔABC, $AB = 7$, $BC = 20$ and $CA = 18$. D and E are points on BC and AC respectively such that $\dfrac{BD}{DC} = \dfrac{BA + AE}{EC}$ and the area of $ABDE$ is equal to the area of ΔCDE. Find the length of CD. (7 marks)

20. 有多少個五位數除以 14、21 和 34 時所得的餘數皆相同？　　　　　　　（7分）

How many five-digit numbers would leave the same remainder when divided by 14, 21 and 34 respectively? (7 marks)

全卷完

END OF PAPER