第十八屆培正數學邀請賽（2019 年）

18th Pui Ching Invitational Mathematics Competition (2019)

初賽（中二組）

Heat Event (Secondary 2)

時限：1 小時 15 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

(a) 本卷共設 20 題，總分為 100 分。

There are 20 questions in this paper and the total score is 100.

(b) 除特別指明外，本卷內的所有數均為十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

(c) 作答時，每題的答案均須以 0 至 9999 之間的整數表示。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

Each answer must be given in the form of an integer between 0 and 9999. Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

(d) 不得使用計算機。

The use of calculators is not allowed.

(e) 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

注意：每題的答案均須以 0 至 9999 之間的整數表示，如有需要應以上述範圍內最接近正確答案的整數回答。如有兩個這樣的整數與正確答案同樣接近，則以「四捨五入」的原則取較大的整數。請細閱答題紙上的指示。

Note: Each answer must be given in the form of an integer between 0 and 9999. Where necessary, the answer should be rounded off to the nearest integer in the above range. Read the instructions on the answer sheet in detail.

1. 現有兩個全等的正四面體，並從每個四面體選一個面然後互相黏合，從而得到一個新的立體。那麼，新的立體有多少條棱？ （3 分）

   Two identical regular tetrahedrons are combined to form a new solid by gluing one face from each tetrahedron to each other. How many edges does the resulting solid have? (3 marks)

2. 現有 n 個不同的兩位正整數，其中每個兩位數都有最少一位數字為 1，另一位數字則為 1、2、3、4 或 5。求 n 的最大可能值。 （3 分）

   There are n different two-digit positive integers. Each two-digit number has at least one digit being 1 and the other digit being 1, 2, 3, 4 or 5. Find the greatest possible value of n. (3 marks)

3. 某等差數列的公差大於 1，且每項均是正整數。若數列的第 2019 項是公差的倍數，則數列中最多可以有多少項是質數？ （3 分）

   There is an arithmetic sequence of positive integers with common difference greater than 1. If the 2019th term of the sequence is a multiple of the common difference, at most how many prime numbers can there be in the sequence? (3 marks)

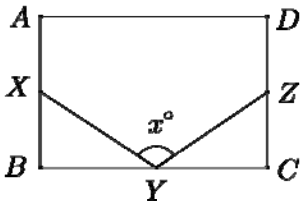
4. 最多可以在一個 7×7 的正方形內互不重疊地放置多少個 2×3 或 3×2 的長方形？ （3 分）

   At most how many 2×3 or 3×2 rectangles can be put in a 7×7 square without overlapping? (3 marks)

5. 現有三個互不相同的正整數，當中任意兩個的最大公因數均不是 1。求這三個整數之和的最小可能值。 （3 分）

   There are three pairwise distinct positive integers. If the H.C.F. of any two of them is not equal to 1, find the smallest possible sum of these three integers. (3 marks)

6. 設 $ABCD$ 為正方形，設 X、Y 和 Z 分別為 AB、BC 和 CD 的中點。若 $\angle XYZ = x°$，求 x 的值。 （4 分）



Let $ABCD$ be a square. Let X, Y and Z be the midpoints of AB, BC and CD respectively. If $\angle XYZ = x°$, find the value of x. (4 marks)

7. 剛開始時，整數 2 和 7 被寫在黑板上。每一步我們將黑板上的兩個數擦掉，並以原來兩數的和與差取而代之。經過 9 步後，黑板上較大的整數是甚麼？ （4 分）

Initially, the integers 2 and 7 are written on the blackboard. In each step, we erase both numbers on the blackboard and replace them by the sum and difference of the two original numbers. What is the larger number on the blackboard after 9 steps? (4 marks)

8. 現有 673 名學生，編號為 1 至 673。他們輪流做一系列共 2019 道題目。首先，編號為 673 的學生解決了其中 $\frac{1}{673}$ 的題目，然後編號為 672 的學生解決了餘下未解決的題目的 $\frac{1}{672}$，如此類推。在編號為 17 的學生做完題目後，還有多少道題目尚未被解決？ （4 分）

There are 673 students labelled from 1 to 673. The students take turns working on a set of 2019 problems. The student with label 673 first solves $\frac{1}{673}$ of the problems, then the student with label 672 solves $\frac{1}{672}$ of the unsolved problems, and so on. How many problems remain unsolved after the student with label 17 has worked on the problems? (4 marks)

9. 若 m 和 n 均為不超過 10 的正整數，且 m^2+mn+n 和 n^2 的奇偶性相同（即同時是奇數或同時是偶數），則有序數對 (m,n) 有多少組不同的可能值？ （5 分）

If m and n are positive integers not exceeding 10 such that m^2+mn+n and n^2 have the same parity (i.e. they are both odd or both even), how many different sets of possible values of the ordered pair (m,n) are there? (5 marks)

10. 非負整數數列 30, 17, 13, … 從第三項開始，每項均等於前兩項之差。求這個數列的首 2019 項之和。（5 分）

In the sequence of non-negative integers 30, 17, 13, …, each term starting from the third is equal to the difference between the two previous terms. Find the sum of the first 2019 terms in the sequence. (5 marks)

11. 假設 ΔABC 和 ΔXYZ 相似，但對應頂點不一定按正確次序表示。若 $AB=3$、$BC=4$、$CA=5$ 及 $XY=60$，求 ΔXYZ 的周長的所有可能值之和。（5 分）

Suppose ΔABC and ΔXYZ are similar but the corresponding vertices are not necessarily shown in the correct order. If $AB=3$, $BC=4$, $CA=5$ and $XY=60$, find the sum of all possible values of the perimeter of ΔXYZ. (5 marks)

12. 在一個遊戲開始時，8 人坐在圍成一圈的 8 張椅子上。現要求每人同時移至一張相鄰的椅子，使得每張椅子依然被一個人佔據著。這些人共有多少種不同的方法進行移動？（6 分）

At the beginning of a game, 8 people are seated around a circle. Now each of them is required to move to an adjacent seat simultaneously, such that each seat remains occupied by one person. In how many different ways can these people move? (6 marks)

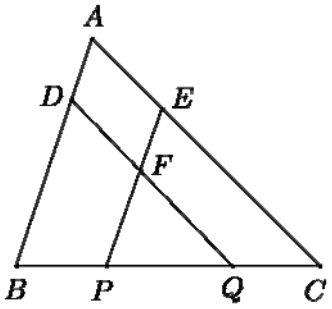
13. 現有一個 5 行 3 列的方格表，每個方格各寫有一個正整數（可以重複）。第一列和第三列的整數都是由上至下嚴格遞增的，第二列的整數是由上至下嚴格遞減的。此外，第三列的每個整數均等於它左邊的兩個數之和。求右下角的方格中的整數的最小可能值。（6 分）

There is a table with 5 rows and 3 columns. A positive integer is written in each cell of the table (possibly with repetition). It is known that the integers in the first column, as well as those in the third column, are strictly increasing from top to bottom, while the integers in the second column are strictly decreasing from top to bottom. Furthermore, each integer in the third column is equal to the sum of the two integers on its left. Find the smallest possible value of the integer in the bottom right corner cell. (6 marks)

14. 有多少個四位數當中任意兩個相鄰數字的奇偶性均不同？ （6 分）

How many four-digit numbers are there such that every two adjacent digits have different parities? (6 marks)

15. 在 ΔABC 中，D 和 E 分別是 AB 和 AC 上的點，使得 $AD:DB=1:5$ 及 $AE:EC=2:3$，而 P 和 Q 則是 BC 上的點，使得 $DQ \parallel AC$ 及 $EP \parallel AB$。假設 F 是 DQ 和 EP 的交點。若 $ADFE$ 的面積是 120，求 ΔFPQ 的面積。 （6 分）



In ΔABC, D and E are points on AB and AC respectively such that $AD:DB=1:5$ and $AE:EC=2:3$, while P and Q are points on BC such that $DQ \parallel AC$ and $EP \parallel AB$. Suppose F is the intersection of DQ and EP. If the area of $ADFE$ is 120, find the area of ΔFPQ. (6 marks)

16. 對任意正整數 k，設 $S(k)$ 表示 k 的數字之和。設 n 是五位數，其五位數字互不相同。求 $S(S(n))$ 的所有可能值之和。 （6 分）

For any positive integer k, let $S(k)$ denote the sum of the digits of k. Let n be a five-digit number whose digits are pairwise distinct. Find the sum of all possible values of $S(S(n))$. (6 marks)

17 現有一行 6 個抽屜。每次我們可以打開其中一個抽屜，然而若一個抽屜有兩個相鄰的抽屜且它們均已經被打開，則我們不能打開這個抽屜。有多少種方法可以逐一打開所有抽屜？ （7 分）

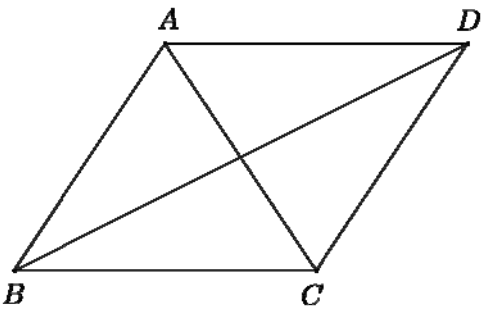
There is a row of 6 drawers. Each time we open one of the drawers. However, we cannot open a drawer if it has two adjacent drawers both of which have been opened. How many ways are there to open all the drawers one by one? (7 marks)

18. 設 a、b、c 為小於 10 的正整數（可以相同）。若存在最少兩個不同的正整數 n，滿足 $\underbrace{\overline{cc\cdots c}}_{2n\text{ 個「}c\text{」}}-\underbrace{\overline{bb\cdots b}}_{n\text{ 個「}b\text{」}}=(\underbrace{\overline{aa\cdots a}}_{n\text{ 個「}a\text{」}})^2$，求三位數 $\overline{abc}$ 的最大可能值。 （7 分）

Let a, b and c be (possibly equal) positive integers less than 10. Suppose there are at least two different positive integers n satisfying $\underbrace{\overline{cc\cdots c}}_{2n\text{ digits}}-\underbrace{\overline{bb\cdots b}}_{n\text{ digits}}=(\underbrace{\overline{aa\cdots a}}_{n\text{ digits}})^2$.

Find the greatest possible value of the three-digit number $\overline{abc}$. (7 marks)

19. 設 $ABCD$ 為平行四邊形，其中 $AD = 32$、$AC = 30$ 及 $BD = 50$。求 AB 的長度。 （7 分）



Let $ABCD$ be a parallelogram with $AD = 32$, $AC = 30$ and $BD = 50$. Find the length of AB. (7 marks)

20. 有多少個五位數除以 14、21 和 34 時所得的餘數皆相同？ （7 分）

How many five-digit numbers would leave the same remainder when divided by 14, 21 and 34 respectively? (7 marks)

全卷完

END OF PAPER