第十七屆培正數學邀請賽（2018 年）

17th Pui Ching Invitational Mathematics Competition (2018)

初賽（高中組）

Heat Event (Senior Secondary)

時限：1 小時 15 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

(a) 本卷共設 20 題，總分為 100 分。

There are 20 questions in this paper and the total score is 100.

(b) 除特別指明外，本卷內的所有數均為十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

(c) 作答時，每題的答案均須以 0 至 9999 之間的整數表示。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

Each answer must be given in the form of an integer between 0 and 9999. Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

(d) 不得使用計算機。

The use of calculators is not allowed.

(e) 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

注意：每題的答案均須以 0 至 9999 之間的整數表示，如有需要應以上述範圍內最接近正確答案的整數回答。如有兩個這樣的整數與正確答案同樣接近，則以「四捨五入」的原則取較大的整數。請細閱答題紙上的指示。

Note: Each answer must be given in the form of an integer between 0 and 9999. Where necessary, the answer should be rounded off to the nearest integer in the above range. Read the instructions on the answer sheet in detail.

1. 設 x 是實數。求 $-2x^2+6x$ 的最大值。（3 分）

   Let x be a real number. Find the maximum value of $-2x^2+6x$. (3 marks)

2. 一個袋中裝有 5 個球，編號分別為 1 至 5。現要從袋中取出兩個球，並規定第二個取出的球的編號必須比第一個的小。那麼，有多少種取球的方法？（3 分）

   There are 5 balls in a bag, labelled 1 to 5. Now two balls are to be picked from the bag, with the requirement that the label of the second chosen ball must be smaller than that of the first one. How many ways are there to pick the balls? (3 marks)

3. 某數列共有 10 項，其中每項均是正整數，且沒有連續三項相等。若該 10 項之和為 S，求 S 的最小可能值。（3 分）

   In a sequence there are 10 terms, each of which is a positive integer. It is given that no three consecutive terms are the same. If the sum of all the 10 terms is S, find the smallest possible value of S. (3 marks)

4. 設 a、b、c 為正整數，使得 $a+b+c=5$。那麼 a^{b^c} 有多少個不同的可能值？（3 分）

   Let a, b and c be positive integers such that $a+b+c=5$. How many different possible values of a^{b^c} are there? (3 marks)

5. 若 $a:b=1:3$ 及 $b:c=2:5$，求 $\frac{c^2}{a^2}$ 的值。（3 分）

   If $a:b=1:3$ and $b:c=2:5$, find the value of $\frac{c^2}{a^2}$. (3 marks)

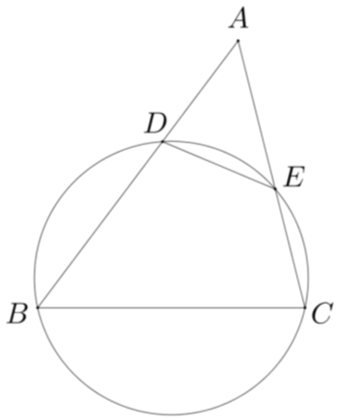
6. 某三角形的其中一隻內角為 80°。若它最小的內角為 x°，求 x 的最大可能值。（3 分）

One of the interior angles of a triangle is 80°. If the smallest interior angle of this triangle is x°, find the greatest possible value of x. (3 marks)

7. 現有 4 張咭片，分別寫上數字 2、3、4 和 5。有多少種不同的方法可以把這些咭片組成一個可被 4 整除的四位數？（4 分）

There are 4 cards with 2, 3, 4, 5 written on them respectively. In how many different ways can these 4 cards be arranged to form a four-digit number divisible by 4? (4 marks)

8. D 和 E 分別是 ΔABC 的邊 AB 和 AC 上的點，使得 B、C、E、D 四點共圓。若 $AD=10$、$AE=14$、$DE=12$ 及 $BC=18$，求 BD 的長度。（4 分）



D and E are points on sides AB and AC of ΔABC respectively such that B, C, E, D are concyclic. If $AD=10$, $AE=14$, $DE=12$ and $BC=18$, find the length of BD. (4 marks)

9. 有多少種方法可以選出最少 3 個不超過 8 且互不相同的正整數？（5 分）

How many ways are there to choose at least 3 positive integers not exceeding 8 which are pairwise distinct? (5 marks)

10. 設 n 為正整數，它有剛好 10 個正因數。求 n 的最小可能值。（5 分）

Let n be a positive integer with exactly 10 positive factors. Find the smallest possible value of n. (5 marks)

11. 設 $f(x)=x^4+ax^3+bx^2+cx+d$，其中 a、b、c、d 為實數。若 $f(x)$ 分別除以 $x+4$、$x-1$、$x+7$ 和 $x+8$ 時所得的餘數均是 2018，求 d 的值。（6 分）

Let $f(x)=x^4+ax^3+bx^2+cx+d$ where a, b, c, d are real numbers. If $f(x)$ leaves the same remainder of 2018 when divided by each of $x+4$, $x-1$, $x+7$ and $x+8$, find the value of d. (6 marks)

12. 方程 $\cos^4 x° - \sin^4 x° = \frac{1}{2}$ 在 $0 \le x \le 360$ 的範圍內有多少個解？ （6 分）

How many solutions are there to the equation $\cos^4 x° - \sin^4 x° = \frac{1}{2}$ in the range $0 \le x \le 360$? (6 marks)

13. 一個鈍角三角形三條邊的長度分別是 10、17 和 x，其中 x 是正整數。求 x 的所有可能值之和。 （6 分）

The side lengths of an obtuse-angled triangle are 10, 17 and x respectively, where x is a positive integer. Find the sum of all possible values of x. (6 marks)

14. 設 $n = 1 \times 2 \times 3 \times \cdots \times 25$。若隨機選取 n 的一個正因數，則這個數是奇數的概率為 p。求 $\frac{1}{p}$ 的值。 （6 分）

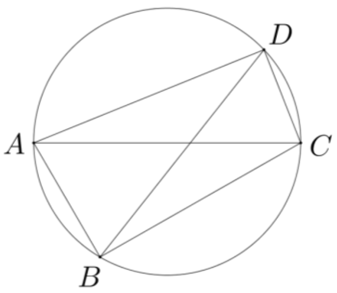
Let $n = 1 \times 2 \times 3 \times \cdots \times 25$. When a positive factor of n is chosen randomly, the probability that the chosen number is odd is p. Find the value of $\frac{1}{p}$. (6 marks)

15. 求方程 $x^3 - x^2 - 31x - 77 = 0$ 所有實根之和。 （6 分）

Find the sum of all real roots to the equation $x^3 - x^2 - 31x - 77 = 0$. (6 marks)

16. 圖中，AC 是圓形的直徑，B、D 是圓周上的兩點。若 $AC = 20$ 及 $BD = 16$，求 $AB \times BC + AD \times DC$ 的最大可能值。 （6 分）

In the figure, AC is a diameter of the circle and B, D are two points on the circumference. If $AC = 20$ and $BD = 16$, find the greatest possible value of $AB \times BC + AD \times DC$. (6 marks)

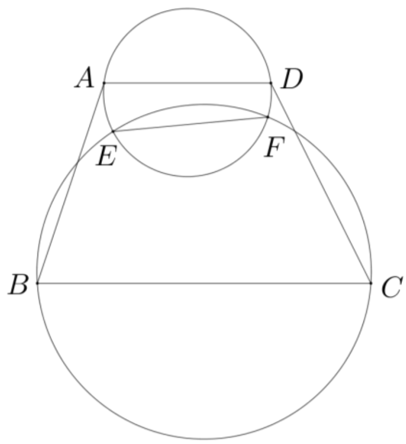


17. 設 n 為正整數，且 d 為 $n^2 + 173$ 和 $3n^2 + 2n + 520$ 的最大公因數。求 d 的最大可能值。 （7 分）

Let n be a positive integer and d be the H.C.F. of $n^2 + 173$ and $3n^2 + 2n + 520$. Find the greatest possible value of d. (7 marks)

18. $ABCD$ 是梯形，其中 $AD \mathbin{/\mkern-5mu/} BC$。兩個分別以 AD 和 BC 為直徑的圓形相交於 E 和 F。若 $EF \mathbin{/\mkern-5mu/} AD$ ，$AD = 50$ 、 $BC = 78$ 及 $EF = 30$ ，求 $ABCD$ 的面積。 （7 分）

$ABCD$ is a trapezium with $AD \mathbin{/\mkern-5mu/} BC$. The two circles with diameters AD and BC respectively meet at E and F. If $EF \mathbin{/\mkern-5mu/} AD$, $AD = 50$, $BC = 78$ and $EF = 30$, find the area of $ABCD$. (7 marks)



19. 某次測驗共設 10 題。沛怡順序回答各題，並答對了第 1 題和答錯了第 2 題。在之後的每一題，沛怡答對的概率均等於她在之前各題答對的比率（例如：因為她在首 2 題答對了 1 題，因此她答對第 3 題的概率為 $\frac{1}{2}$ ）。設 p 為沛怡在 10 題中答對剛好 5 題的概率。求 $\frac{1}{p}$ 的值。 （7 分）

There were 10 questions in a test. Emily answered the questions in order, and she got Question 1 correct and Question 2 wrong. For each subsequent question, the probability for Emily to answer correctly is equal to the proportion of correct answers in the previous questions. (For example, since Emily got 1 correct answer in the first 2 questions, the probability that she could answer Question 3 correctly is $\frac{1}{2}$.) Let p be the probability that Emily got exactly 5 questions out of 10 questions correct. Find the value of $\frac{1}{p}$. (7 marks)

20. 求 $100(\cos 6^\circ + \cos 66^\circ + \cos 78^\circ + \cos 138^\circ)^2$ 的值。 （7 分）

Find the value of $100(\cos 6^\circ + \cos 66^\circ + \cos 78^\circ + \cos 138^\circ)^2$. (7 marks)

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END OF PAPER