第十七屆培正數學邀請賽（2018 年）

17th Pui Ching Invitational Mathematics Competition (2018)

初賽（中四組）

Heat Event (Secondary 4)

時限：1 小時 15 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

(a) 本卷共設 20 題，總分為 100 分。

There are 20 questions in this paper and the total score is 100.

(b) 除特別指明外，本卷內的所有數均為十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

(c) 作答時，每題的答案均須以 0 至 9999 之間的整數表示。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

Each answer must be given in the form of an integer between 0 and 9999. Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

(d) 不得使用計算機。

The use of calculators is not allowed.

(e) 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

注意：每題的答案均須以 0 至 9999 之間的整數表示，如有需要應以上述範圍內最接近正確答案的整數回答。如有兩個這樣的整數與正確答案同樣接近，則以「四捨五入」的原則取較大的整數。請細閱答題紙上的指示。

Note: Each answer must be given in the form of an integer between 0 and 9999. Where necessary, the answer should be rounded off to the nearest integer in the above range. Read the instructions on the answer sheet in detail.

1. 設 x 是實數。求 $-2x^2+6x$ 的最大值。（3 分）

Let x be a real number. Find the maximum value of $-2x^2+6x$. (3 marks)

2. 一個袋中裝有 5 個球，編號分別為 1 至 5。現要從袋中取出兩個球，並規定第二個取出的球的編號必須比第一個的小。那麼，有多少種取球的方法？（3 分）

There are 5 balls in a bag, labelled 1 to 5. Now two balls are to be picked from the bag, with the requirement that the label of the second chosen ball must be smaller than that of the first one. How many ways are there to pick the balls? (3 marks)

3. 若某正整數的每位數字（除首位數字外）均大於它左方的數字，則我們稱這個正整數是「遞增數」，例如：1234 和 2357 均是「遞增數」。若 n 是偶數並且是大於 2018 的「遞增數」，求 n 的最小可能值。（3 分）

A positive integer is said to be ‘ascending’ if each digit (except the first) is larger than the one on its left. For example, 1234 and 2357 are ‘ascending’ integers. If n is even and is an ‘ascending’ integer greater than 2018, find the smallest possible value of n. (3 marks)

4. 最多可在一個 5×5 正方形內不重疊地放置多少個 2×2 正方形？（3 分）

At most how many 2×2 squares can be placed inside a 5×5 square without overlapping? (3 marks)

5. 若 $a:b=1:3$ 及 $b:c=2:5$，求 $\frac{c^2}{ab}$ 的值。（3 分）

If $a:b=1:3$ and $b:c=2:5$, find the value of $\frac{c^2}{ab}$. (3 marks)

6. 設 p 和 q 為小於 10 的質數。求 $p\times q$ 的所有可能值之和。（4 分）

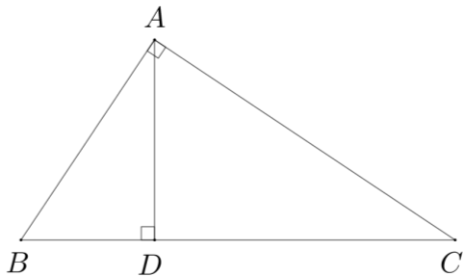
Let p and q be prime numbers less than 10. Find the sum of all possible values of $p\times q$. (4 marks)

7. 現有 52 張咭，其中編號為 1、2、3、⋯、13 的咭各有 4 張，且每張咭只有一個編號。求 n 的最大可能值，使得以下命題成立：「若隨機選取 13 張咭，當中必定存在編號不小於 n 的咭」。（4 分）

There are 52 cards. Each of the labels 1, 2, 3, …, 13 appears on 4 of the cards, and each card has exactly one label. Find the greatest possible value of n for which the following statement holds: 'if 13 cards are chosen at random, there must be a card whose label is not less than n'. (4 marks)

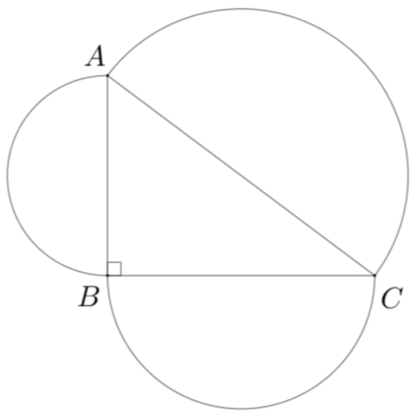
8. 在 ΔABC 中，$\angle BAC = 90°$，D 為 A 到 BC 的垂足。若 $BD = 12$ 及 $DC = 27$，求 AD 的長度。（4 分）

In ΔABC, $\angle BAC = 90°$ and D is the foot of perpendicular from A to BC. If $BD = 12$ and $DC = 27$, find the length of AD. (4 marks)



9. 在 ΔABC 中，$\angle ABC = 90°$。以 AB 和 BC 為直徑的半圓的面積分別是 9 和 12。求以 AC 為直徑的半圓的面積。（5 分）

In ΔABC, $\angle ABC = 90°$. The areas of the semi-circles with diameters AB and BC are 9 and 12 respectively. Find the area of the semi-circle with diameter AC. (5 marks)



10. 對正整數 m，定義 $m! = 1\times 2\times\cdots\times m$。若 n 是正整數，且 $(n!)!$ 是 10^{2018} 的倍數，求 n 的最小可能值。（5 分）

We define $m! = 1\times 2\times\cdots\times m$ for positive integer m. If n is a positive integer such that $(n!)!$ is a multiple of 10^{2018}, find the smallest possible value of n. (5 marks)

11. 有多少種方法可以選出最少 3 個不超過 8 且互不相同的正整數？（5 分）

How many ways are there to choose at least 3 positive integers not exceeding 8 which are pairwise distinct? (5 marks)

12. 設 $f(x)=x^4+ax^3+bx^2+cx+d$，其中 a、b、c、d 為實數。若 $f(x)$ 分別除以 $x+4$、$x-1$、$x+7$ 和 $x+8$ 時所得的餘數均是 2018，求 d 的值。（6 分）

Let $f(x)=x^4+ax^3+bx^2+cx+d$ where a, b, c, d are real numbers. If $f(x)$ leaves the same remainder of 2018 when divided by each of $x+4$, $x-1$, $x+7$ and $x+8$, find the value of d. (6 marks)

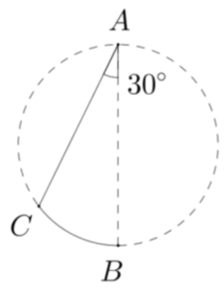
13. 方程 $\cos^6 x°-\sin^6 y°=1$ 在 $0\le x, y\le 540$ 的範圍內有多少個解？（6 分）

How many solutions are there to the equation $\cos^6 x°-\sin^6 y°=1$ in the range $0\le x, y\le 540$? (6 marks)

14. 求方程 $x^3-x^2-31x-77=0$ 所有實根之和。（6 分）

Find the sum of all real roots to the equation $x^3-x^2-31x-77=0$. (6 marks)

15. 圖中所示為一個半徑為 $12\sqrt{3}$ 米的圓形游泳池。AB 是泳池的直徑，C 是圓周上的一點使得 $\angle CAB=30°$。志文打算從 A 出發，首先以直線游至 C，然後沿着劣弧走至 B。若志文游泳的速度是每秒 1 米，在圓周上行走的速度是每秒 $\sqrt{3}\pi$ 米，則他從 A 到 B 共需多少秒？（6 分）



The figure shows a circular swimming pool with radius $12\sqrt{3}$ metres. AB is a diameter of the circle, and C is a point on the circumference such that $\angle CAB=30°$. Herman plans to start from A, swim along a straight line to C, and then walk along the minor arc to B. Suppose the swimming speed of Herman is 1 m/s and his walking speed along the circumference is $\sqrt{3}\pi$ m/s. How many seconds would he need to go from A to B? (6 marks)

16. 設 $f(x)=\sqrt{x^2-2x+2}+\sqrt{x^2-32x+305}$。若 $f(x)$ 在 $x=a$ 時達至最小值，求 $100a$ 的值。（6 分）

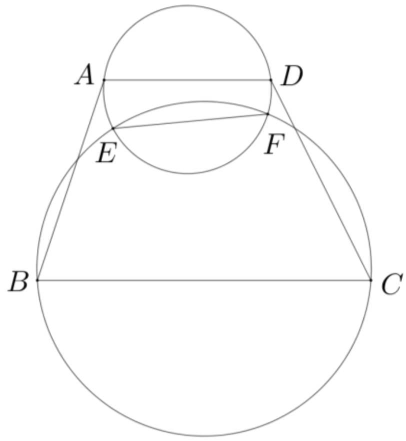
Let $f(x)=\sqrt{x^2-2x+2}+\sqrt{x^2-32x+305}$. If $f(x)$ attains its minimum value when $x=a$, find the value of $100a$. (6 marks)

17. 設 a、b、p 為整數，其中 $a+b\geq 0$ 且 p 是質數。若 $a^3+b^3-6ab=p-8$，求 p 的所有可能值之和。（7 分）

Let a, b, p be integers such that $a+b\geq 0$ and p is prime. If $a^3+b^3-6ab=p-8$, find the sum of all possible values of p. (7 marks)

18. $ABCD$ 是梯形，其中 AD // BC。兩個分別以 AD 和 BC 為直徑的圓形相交於 E 和 F。若 EF // AD，$AD=50$、$BC=78$ 及 $EF=30$，求 $ABCD$ 的面積。（7 分）

$ABCD$ is a trapezium with AD // BC. The two circles with diameters AD and BC respectively meet at E and F. If EF // AD, $AD=50$, $BC=78$ and $EF=30$, find the area of $ABCD$. (7 marks)



19. 某次測驗共設 10 題。沛怡順序回答各題，並答對了第 1 題和答錯了第 2 題。在之後的每一題，沛怡答對的概率均等於她在之前各題答對的比率（例如：因為她在首 2 題答對了 1 題，因此她答對第 3 題的概率為 $\frac{1}{2}$）。設 p 為沛怡在 10 題中答對剛好 5 題的概率。求 $\frac{1}{p}$ 的值。（7 分）

There were 10 questions in a test. Emily answered the questions in order, and she got Question 1 correct and Question 2 wrong. For each subsequent question, the probability for Emily to answer correctly is equal to the proportion of correct answers in the previous questions. (For example, since Emily got 1 correct answer in the first 2 questions, the probability that she could answer Question 3 correctly is $\frac{1}{2}$.) Let p be the probability that Emily got exactly 5 questions out of 10 questions correct. Find the value of $\frac{1}{p}$. (7 marks)

20. 設 x 和 y 為實數。若 $|x+y| \geq |1-xy|$，求 $\dfrac{100|x+y|}{\sqrt{(x^2+1)(y^2+1)}}$ 的最小可能值。（7 分）

Let x and y be real numbers. If $|x+y| \geq |1-xy|$, find the smallest possible value of $\dfrac{100|x+y|}{\sqrt{(x^2+1)(y^2+1)}}$. (7 marks)

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END OF PAPER