第十七屆培正數學邀請賽（2018 年）

17th Pui Ching Invitational Mathematics Competition (2018)

初賽（中三組）

Heat Event (Secondary 3)

時限：1 小時 15 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

(a) 本卷共設 20 題，總分為 100 分。

There are 20 questions in this paper and the total score is 100.

(b) 除特別指明外，本卷內的所有數均為十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

(c) 作答時，每題的答案均須以 0 至 9999 之間的整數表示。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

Each answer must be given in the form of an integer between 0 and 9999. Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

(d) 不得使用計算機。

The use of calculators is not allowed.

(e) 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

注意：每題的答案均須以 0 至 9999 之間的整數表示，如有需要應以上述範圍內最接近正確答案的整數回答。如有兩個這樣的整數與正確答案同樣接近，則以「四捨五入」的原則取較大的整數。請細閱答題紙上的指示。

Note: Each answer must be given in the form of an integer between 0 and 9999. Where necessary, the answer should be rounded off to the nearest integer in the above range. Read the instructions on the answer sheet in detail.

1. 若某正整數的每位數字（除首位數字外）均大於它左方的數字，則我們稱這個正整數是「遞增數」，例如：1234 和 2357 均是「遞增數」。若 n 是大於 2018 的「遞增數」，求 n 的最小可能值。（3 分）

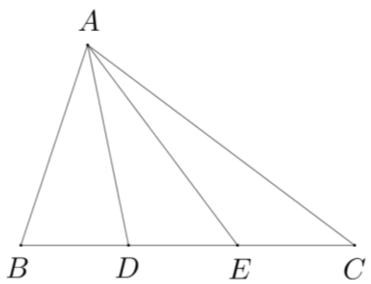
   A positive integer is said to be ‘ascending’ if each digit (except the first) is larger than the one on its left. For example, 1234 and 2357 are ‘ascending’ integers. If n is an ‘ascending’ integer greater than 2018, find the smallest possible value of n. (3 marks)

2. 設 x、y 為實數，使得 $\dfrac{3x+y}{x-3y}=-2$。求 $\dfrac{x+3y}{3x-y}$ 的值。（3 分）

   Let x and y be real numbers such that $\dfrac{3x+y}{x-3y}=-2$. Find the value of $\dfrac{x+3y}{3x-y}$. (3 marks)

3. 在 ΔABC 中，點 D 和 E 在邊 BC 上，使得 $BD=DE=EC$。若 ΔABE 的面積是 36，求 ΔACD 的面積。（3 分）

   In ΔABC, points D and E lie on side BC such that $BD=DE=EC$. If the area of ΔABE is 36, find the area of ΔACD. (3 marks)



4. 最多可在一個 5×5 正方形內不重疊地放置多少個 2×2 正方形？（3 分）

   At most how many 2×2 squares can be placed inside a 5×5 square without overlapping? (3 marks)

5. 一顆骰子的 6 個面各寫有一個正整數（可以相同）。若任意兩個相鄰面的整數均不相同，求這 6 個正整數之和的最小可能值。（4 分）

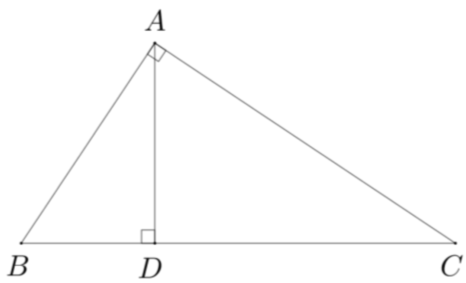
A positive integer is written on each of the 6 faces of a die (the integers can be the same). If the integers on any two adjacent faces must be different, find the smallest possible value of the sum of these 6 positive integers. (4 marks)

6. 現有 52 張咭，其中編號為 1、2、3、⋯、13 的咭各有 4 張，且每張咭只有一個編號。求 n 的最小可能值，使得以下命題成立：「若隨機選取 13 張咭，當中必定存在編號不大於 n 的咭」。（4 分）

There are 52 cards. Each of the labels 1, 2, 3, …, 13 appears on 4 of the cards, and each card has exactly one label. Find the smallest possible value of n for which the following statement holds: ‘if 13 cards are chosen at random, there must be a card whose label is not greater than n’. (4 marks)

7. 在 ΔABC 中，$\angle BAC = 90°$，D 為 A 到 BC 的垂足。若 $BD = 12$ 及 $DC = 27$，求 AD 的長度。（4 分）

In ΔABC, $\angle BAC = 90°$ and D is the foot of perpendicular from A to BC. If $BD = 12$ and $DC = 27$, find the length of AD. (4 marks)



8. 有多少個不同的三角形每邊的長度均是 1、3 或 5？（若兩個三角形全等，則視為同一個三角形。）（4 分）

How many different triangles are there such that the length of each side is 1, 3 or 5? (If two triangles are congruent, they are regarded to be the same triangle.) (4 marks)

9. 大小相同的正 n 邊形（其中 $n \geq 3$）可用作密鋪座標平面，使得原點附近沒有空隙亦沒有互相重疊的情況。若一個正 n 邊形的其中一個頂點位於原點，求 n 的所有可能值之和。（5 分）

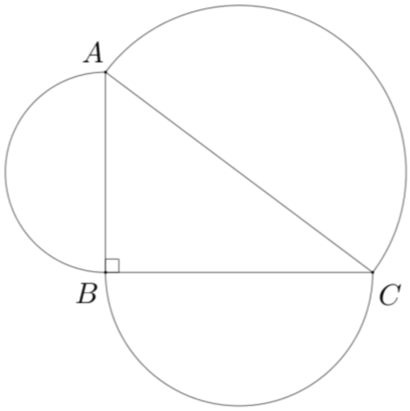
Identical regular n-sided polygons (where $n \geq 3$) can be used to tessellate the coordinate plane in a way such that there is no overlapping and no empty space around the origin. If one of the vertices of a regular n-sided polygon lies on the origin, find the sum of all possible values of n. (5 marks)

10. 對正整數 m，定義 $m!=1\times 2\times\cdots\times m$。若 n 是正整數，且 $(n!)!$ 是 10^{2018} 的倍數，求 n 的最小可能值。 （5 分）

We define $m!=1\times 2\times\cdots\times m$ for positive integer m. If n is a positive integer such that $(n!)!$ is a multiple of 10^{2018}, find the smallest possible value of n. (5 marks)

11. 在 ΔABC 中，$\angle ABC=90^\circ$。以 AB 和 BC 為直徑的半圓的面積分別是 9 和 12。求以 AC 為直徑的半圓的面積。 （5 分）

In ΔABC, $\angle ABC=90^\circ$. The areas of the semi-circles with diameters AB and BC are 9 and 12 respectively. Find the area of the semi-circle with diameter AC. (5 marks)



12. 現有三疊咭片：第一疊有 5 張，編號為 1 至 5；第二疊有 4 張，編號為 1 至 4；第三疊有 3 張，編號為 1 至 3。有多少種方法從每疊咭片中各選一張，使得三張被選的咭的編號互不相同？ （5 分）

There are three piles of cards. The first pile consists of 5 cards labelled 1 to 5; the second pile consists of 4 cards labelled 1 to 4; the third pile consists of 3 cards labelled 1 to 3. How many ways are there to choose one card from each pile so that the labels of the three chosen cards are pairwise distinct? (5 marks)

13. 設 $f(x,y)=x^2-2xy+3y^2$。在以下四個命題中，求所有真命題的編號之和。

(1) 對於任意實數 x 和 y，若 $f(x,y)=1$，則 $[f(x,y)]^2=1$。

(2) 對於任意實數 x 和 y，若 $f(x,y)=-1$，則 $[f(x,y)]^2=-1$。

(4) 對於任意實數 x 和 y，若 $f(x,y)=1$，則 $[f(x,y)]^3=1$。

(8) 對於任意實數 x 和 y，若 $f(x,y)=-1$，則 $[f(x,y)]^3=-1$。 （6 分）

Let $f(x,y)=x^2-2xy+3y^2$. Among the following statements, find the sum of the labels of all the true statements.

(1) For any real numbers x and y, if $f(x,y)=1$, then $[f(x,y)]^2=1$.

(2) For any real numbers x and y, if $f(x,y)=-1$, then $[f(x,y)]^2=-1$.

(4) For any real numbers x and y, if $f(x,y)=1$, then $[f(x,y)]^3=1$.

(8) For any real numbers x and y, if $f(x,y)=-1$, then $[f(x,y)]^3=-1$. (6 marks)

14. 在一個實數數列中，從第二項起每項均比上一項的立方大 1。若數列的第 2018 項是 2018，求它的第 2016 項，答案準確至最接近整數。（6 分）

In a sequence of real numbers, each term from the second is 1 greater than the cube of the previous term. If the 2018th term of the sequence is 2018, find its 2016th term correct to the nearest integer. (6 marks)

15. 某次測驗共設 10 道是非題。若答案為「是」的題目總數比答案為「否」的多，那麼這次測驗的答案共有多少種不同的可能性？（6 分）

There are 10 true or false questions in a test. If there are more questions whose answer is ‘True’ than those whose answer is ‘False’, how many different possibilities are there for the answers of this test? (6 marks)

16. 設 $f(x)=\sqrt{x^2-2x+2}+\sqrt{x^2-32x+305}$，若 $f(x)$ 在 $x=a$ 時達至最小值，求 $100a$ 的值。（6 分）

Let $f(x)=\sqrt{x^2-2x+2}+\sqrt{x^2-32x+305}$. If $f(x)$ attains its minimum value when $x=a$, find the value of $100a$. (6 marks)

17. 設 m 和 n 為正整數。把 $\frac{m}{n}$ 以小數表示時，小數點後的首三個位從左至右為 2、9、4。求 $m+n$ 的最小可能值。（7 分）

Let m and n be positive integers. When $\frac{m}{n}$ is expressed as a decimal, the first three digits after the decimal point are (from left to right) 2, 9, 4. Find the smallest possible value of $m+n$. (7 marks)

18. 在一個數列中，每項均為正整數。從第三項開始，每項均等於前兩項之和。若數列的第 n 項是 2018，求 n 的最大可能值。（7 分）

In a sequence, each term is a positive integer. Starting from the third term, each term is equal to the sum of the previous two terms. If the n-th term of the sequence is 2018, find the greatest possible value of n. (7 marks)

19. 設 a、b、p 為整數，其中 $a+b\geq 0$ 且 p 是質數。若 $a^3+b^3-6ab=p-8$，求 p 的所有可能值之和。 （7 分）

Let a, b, p be integers such that $a+b\geq 0$ and p is prime. If $a^3+b^3-6ab=p-8$, find the sum of all possible values of p. (7 marks)

20. 在一場魔術表演中，魔術師在平面上選定三個不同的點 A、B、C，然後他邀請一名觀眾在同一平面上選取任意一點 P。接着，魔術師將 P 繞 A 逆時針旋轉 83°，得到點 Q；又將 P 繞 B 順時針旋轉 135°，得到點 R。魔術師可以保證 CQ 和 CR 的長度必定相等。若 $\angle ACB=x^\circ$，求 x 的值。 （7 分）

In a magic show, the magician marked three distinct points A, B and C on the plane. Then he invited an audience to mark an arbitrary point P on the same plane. Next, the magician rotated P about A anti-clockwise by 83° to obtain the point Q, and rotated P about B clockwise by 135° to obtain the point R. The magician could guarantee the lengths of CQ and CR would always be the same. If $\angle ACB=x^\circ$, find the value of x. (7 marks)

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END OF PAPER