第十七屆培正數學邀請賽（2018 年）

17th Pui Ching Invitational Mathematics Competition (2018)

初賽（中二組）

Heat Event (Secondary 2)

時限：1 小時 15 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

(a) 本卷共設 20 題，總分為 100 分。

There are 20 questions in this paper and the total score is 100.

(b) 除特別指明外，本卷內的所有數均為十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

(c) 作答時，每題的答案均須以 0 至 9999 之間的整數表示。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

Each answer must be given in the form of an integer between 0 and 9999. Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

(d) 不得使用計算機。

The use of calculators is not allowed.

(e) 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

注意：每題的答案均須以 0 至 9999 之間的整數表示，如有需要應以上述範圍內最接近正確答案的整數回答。如有兩個這樣的整數與正確答案同樣接近，則以「四捨五入」的原則取較大的整數。請細閱答題紙上的指示。

Note: Each answer must be given in the form of an integer between 0 and 9999. Where necessary, the answer should be rounded off to the nearest integer in the above range. Read the instructions on the answer sheet in detail.

1. 求滿足 $n^2 > n$ 的最小正整數 n。（3 分）

   Find the smallest positive integer n satisfying $n^2 > n$. (3 marks)

2. 現要在算式「4□3□2□1」中的每個空格填入「+」、「−」、「×」或「÷」（可以重複使用）。求算式結果的最大可能值。（3 分）

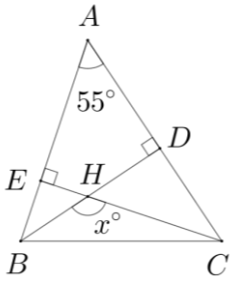
   Each of the squares in the expression '4□3□2□1' is to be filled with one of the symbols '+', '−', '×' and '÷' (repetition is allowed). Find the greatest possible value of the result of the expression. (3 marks)

3. 方程 $ab + c = 5$ 有多少組正整數解 (a, b, c)？（3 分）

   How many sets of positive integer solutions (a, b, c) are there to the equation $ab + c = 5$? (3 marks)

4. ABC 是銳角三角形，其中 $\angle BAC = 55°$。三角形的高 BD 和 CE 相交於 H。若 $\angle BHC = x°$，求 x 的值。（3 分）

   ΔABC is acute-angled with $\angle BAC = 55°$. The altitudes BD and CE of the triangle intersect at H. If $\angle BHC = x°$, find the value of x. (3 marks)



5. 有多少個質數的數字之和是 4，當中所有數字均非零？（4 分）

   How many prime numbers have sum of digits 4 in which each digit is non-zero? (4 marks)

6. 某三角形三邊的長度 a、b 和 c 均為整數。若三角形的周長為 2018，求 a 的最大可能值。（4 分）

   A triangle has side lengths a, b and c, all of which are integers. If the perimeter of the triangle is 2018, find the greatest possible value of a. (4 marks)

7. 有多少個兩位正整數的個位數字小於其十位數字？ （4 分）

How many two-digit positive integers have their unit digit smaller than the tens digit? (4 marks)

8. 設 a、b、c、d 是 2、3、4、5 的某個排列。求 $a^b - c^d$ 的最大可能值。 （4 分）

Suppose a, b, c, d is a permutation of 2, 3, 4, 5. Find the greatest possible value of $a^b - c^d$. (4 marks)

9. 某正整數 n 的數字之和是 2018。那麼 $n+1$ 的數字之和最小是多少？ （5 分）

A positive integer n has sum of digits 2018. What is the smallest possible sum of digits of $n+1$? (5 marks)

10. 安琪須在八月份完成 6 份暑期作業。她會按次序做作業，而且每完成一份作業後，最少休息一整天才開始做下一份作業（例如：若她在 8 月 8 日完成第一份作業，她最少會休息到 8 月 10 日才開始做第二份作業）。在安琪的計劃中，所需時間最短的一份暑期作業要花 n 天才能完成，求 n 的最大可能值。 （5 分）

Angie needs to complete 6 summer holiday assignments in August. She will work on the assignments in order, and, after finishing an assignment, she will take a break of at least one whole day before working on the next. (For example, if she finishes the first assignment on 8th August, she will take a break until at least 10th August before working on the second.) In Angie's plan, the assignment that takes the shortest time needs n days to complete. Find the greatest possible value of n. (5 marks)

11. 某數學比賽的試卷共設 n 道題，分別佔 1^3、2^3、…、n^3 分。若試卷的滿分是 1296，求 n 的值。 （5 分）

In a mathematics competition, the paper consists of n questions that are worth 1^3, 2^3, …, n^3 marks respectively. If the full score of the paper is 1296, find the value of n. (5 marks)

12. 現有一行 10 個抽屜，其中只有兩個相鄰的抽屜各有一本書，其餘抽屜則是空的。最少需要逐一打開多少個抽屜，才可保證能把兩本書都拿出來？ （6 分）

There are 10 drawers in a row. Among them, only two consecutive drawers each contains a book, while the rest are empty. At least how many drawers should be opened one by one to ensure that we can take out both books? (6 marks)

13. 設 n 是正整數，使得 $n^{1800}=60^n$ 。求 n 的值。（6 分）

Let n be a positive integer such that $n^{1800}=60^n$. Find the value of n. (6 marks)

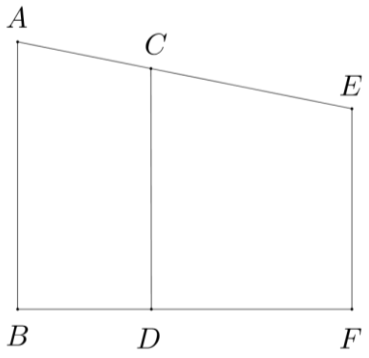
14. 某個五邊形的其中四隻內角為 100°、100°、110° 和 120°。現要於五邊形的每條邊各寫下一個整數，它等於這條邊的兩隻相鄰角之和（以「度」為單位）。這 5 個整數有多少組不同的可能值？（如果從某一組可能值通過旋轉五邊形可得到另一組可能值，則這兩組可能值視為相同。）（6 分）

Four of the interior angles of a pentagon are 100°, 100°, 110° and 120°. Now on each side of the pentagon we write down an integer equal to the sum (in degrees) of the two angles adjacent to the side. How many sets of possibilities are there for these 5 integers? (Two sets of values are regarded to be the same if one could be obtained from the other by rotating the pentagon.) (6 marks)

15. 設 n 是正整數。若 n 的第二大的因數是 35，求 n 的所有可能值之和。（6 分）

Let n be a positive integer. If the second largest factor of n is 35, find the sum of all possible values of n. (6 marks)

16. 圖中，AB、CD 和 EF 互相平行，其中 A、C、E 共線，而 B、D、F 亦共線。若 $AC=8$、$CE=16$、$AB=20$ 及 $CD=18$，求 EF 的長度。（6 分）



In the figure, AB, CD and EF are parallel to each other, with A, C, E being collinear and B, D, F also being collinear. If $AC=8$, $CE=16$, $AB=20$ and $CD=18$, find the length of EF. (6 marks)

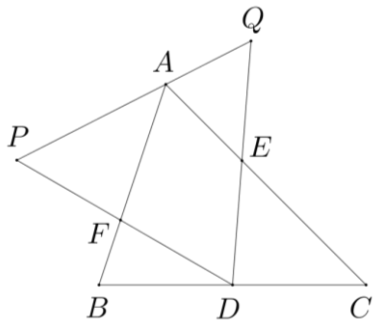
17. 求 $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots+\frac{1}{32}$ 的值，答案準確至最接近整數。（6 分）

Find the value of $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots+\frac{1}{32}$ correct to the nearest integer. (6 marks)

18. 現有一個 5×5 的棋盤。小莊把其中 k 個方格塗上黑色，其餘的方格則塗上白色。媽媽和小莊在棋盤上進行以下遊戲：媽媽把一枚棋子放在棋盤左下角的方格，然後每次把棋子向上或向右移動一格，最後到達棋盤右上角的方格。每次移動棋子後，媽媽都會告訴小莊棋子當時位處的方格的顏色。事後小莊需要猜出棋子的移動路線。若小莊總能成功，求 k 的最大可能值。 （7 分）

There is a 5×5 chessboard. John colours k of the cells black and the rest of them white. He plays a game with his mother on the chessboard as follows. Mum puts a chess piece in the bottom left corner cell of the chessboard. She then moves the chess piece to the cell immediately above or immediately to the right, until reaching the upper right corner cell. Every time when the chess piece is moved, Mum will tell John the colour of the cell in which the chess piece lands. After the piece reaches the end, John will have to guess the route taken by the chess piece afterwards. If John can always succeed, find the greatest possible value of k. (7 marks)

19. 在 ΔABC 中，F 是 AB 上的一點使得 $AF:FB=3:1$，E 是 AC 上的一點使得 $AE:EC=1:2$，而 D 則是 BC 上的一點。設 P 為一點使得 F 是 DP 的中點，而 Q 為一點使得 E 是 DQ 的中點。若 P、A、Q 共線，求 $360\times\frac{BD}{DC}$ 的值。 （7 分）



In ΔABC, F is a point on AB such that $AF:FB=3:1$, E is a point on AC such that $AE:EC=1:2$ and D is a point on BC. Let P be the point such that F is the midpoint of DP, and Q be the point such that E is the midpoint of DQ. If P, A and Q are collinear, find the value of $360\times\frac{BD}{DC}$. (7 marks)

20. 某班有 80 名學生，他們的學號分別為 1 至 80。老師把他們分成 k 組，當中 k 大於 1，使得每組的學生數目相同，且每組中所有學生的學號之和亦相同。求 k 的所有可能值之和。（7 分）

In a class there are 80 students and they are assigned class numbers from 1 to 80. The teacher divided them into k groups, where k is greater than 1, each containing the same number of students. Furthermore, the sum of class numbers of the members is the same for each group. Find the sum of all possible values of k. (7 marks)

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END OF PAPER