

第十七屆培正數學邀請賽（2018 年）

17th Pui Ching Invitational Mathematics Competition (2018)

初賽（中一組）

Heat Event (Secondary 1)

時限：1 小時 15 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

- (a) 本卷共設 20 題，總分為 100 分。

There are 20 questions in this paper and the total score is 100.

- (b) 除特別指明外，本卷內的所有數均為十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

- (c) 作答時，每題的答案均須以 0 至 9999 之間的整數表示。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

Each answer must be given in the form of an integer between 0 and 9999. Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

- (d) 不得使用計算機。

The use of calculators is not allowed.

- (e) 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

注意：每題的答案均須以 0 至 9999 之間的整數表示，如有需要應以上述範圍內最接近正確答案的整數回答。如有兩個這樣的整數與正確答案同樣接近，則以「四捨五入」的原則取較大的整數。請細閱答題紙上的指示。

Note: Each answer must be given in the form of an integer between 0 and 9999. Where necessary, the answer should be rounded off to the nearest integer in the above range. Read the instructions on the answer sheet in detail.

1. 求最小的四位數，當中所有數字均是非零合成數。 (3 分)

Find the smallest four-digit number in which each digit is a non-zero composite number. (3 marks)

2. 設 a 、 b 、 c 、 d 為整數。若 $a+b=10$ 、 $b+c=18$ 及 $c+d=20$ ，求 $a+d$ 的值。 (3 分)

Let a, b, c and d be integers. If $a+b=10$, $b+c=18$ and $c+d=20$, find the value of $a+d$. (3 marks)

3. 一間商店規定不設找續。小賓帶著四枚面值分別為 1 元、2 元、5 元和 10 元的硬幣到該店購物。他打算購買一件價值 n 元的貨品，當中 n 是正整數，但發現自己無法付款剛好 n 元。求 n 的最小可能值。 (3 分)

A shop accepts exact payment only. Benny went to the shop with four coins whose denominations are 1 dollar, 2 dollars, 5 dollars and 10 dollars respectively. He planned to buy an item worth n dollars where n is a positive integer, but found that he was unable to make the exact payment. Find the smallest possible value of n . (3 marks)

4. 兩塊三角形的咭紙可拼成一個 n 邊形，其中兩塊咭紙沒有重疊。求 n 的最小可能值。 (3 分)

Two pieces of triangular cardboard can be combined to form an n -sided polygon with no overlapping between the two pieces. Find the smallest possible value of n . (3 marks)

5. 設 n 是正整數。若 n 的第二大的因數是 12，求 n 的值。 (3 分)

Let n be a positive integer. If the second largest factor of n is 12, find the value of n . (3 marks)

6. 某等腰三角形的其中一隻內角為 56° ，其最大的內角則是 x° 。求 x 的所有可能值之和。(4 分)

One of the interior angles of an isosceles triangle is 56° , while its largest interior angle is x° . Find the sum of all possible values of x . (4 marks)

7. 某三角形三邊的長度 a 、 b 和 c 均為整數。若三角形的周長為 2018，求 a 的最大可能值。(4 分)

A triangle has side lengths a , b and c , all of which are integers. If the perimeter of the triangle is 2018, find the greatest possible value of a . (4 marks)

8. 某數學比賽的試卷共設 n 道題，分別佔 1、3、5、 \dots 分（從第二題起每題的佔分均比之前一題多 2 分）。若試卷的滿分是 100，求 n 的值。(4 分)

In a mathematics competition, the paper consists of n questions that are worth 1, 3, 5, ... marks respectively (starting from the second question each question is worth 2 more marks than the previous question). If the full score of the paper is 100, find the value of n . (4 marks)

9. 現有一行 3 個方格。第一和第三個方格可分別填上 1 至 9 其中一個整數（可以重複使用），第二個方格則可填上「 $>$ 」、「 $=$ 」或「 $<$ 」其中一個符號。有多少種填方格的方法可以得出正確的關係式？(5 分)

There are 3 cells in a row. In each of the first and the third cells, one of the integers from 1 to 9 is filled in (repetition is allowed). In the second cell, one of the symbols ' $>$ ', ' $=$ ' and ' $<$ ' is filled in. How many ways are there to fill in the cells to give a correct relation? (5 marks)

10. 一隻草蜢沿著一條直線跳躍。每次跳躍可以是向前或向後跳，距離均是 2 的冪（即 1、2、4、8、 \dots ），而每次跳躍的距離可以相同亦可以不同。那麼這隻草蜢最少需要跳躍多少次，才可到達與起點距離 2018 的地方？(5 分)

A grasshopper jumps along a straight line. Each time it jumps forward or backward a distance which is equal to a power of 2 (i.e. 1, 2, 4, 8, ...), and the distances jumped may or may not be the same. What is the minimum number of jumps needed so that the grasshopper can land at a distance of 2018 from its initial position? (5 marks)

11. 某個五邊形的其中四隻內角為 100° 、 100° 、 110° 和 120° 。現要於五邊形的每條邊各寫下一個整數，它等於這條邊的兩隻相鄰角之和（以「度」為單位）。這 5 個整數有多少組不同的可能值？（如果從某一組可能值通過旋轉五邊形可得到另一組可能值，則這兩組可能值視為相同。）（6 分）

Four of the interior angles of a pentagon are 100° , 100° , 110° and 120° . Now on each side of the pentagon we write down an integer equal to the sum (in degrees) of the two angles adjacent to the side. How many sets of possibilities are there for these 5 integers? (Two sets of values are regarded to be the same if one could be obtained from the other by rotating the pentagon.) (6 marks)

12. 設 n 是正整數，使得 $n^{1800} = 60^n$ 。求 n 的值。（6 分）

Let n be a positive integer such that $n^{1800} = 60^n$. Find the value of n . (6 marks)

13. 最少要有多少張邊長為 1 的等邊三角形咭紙，才可拼成一個凸五邊形（咭紙不可互相重疊）？（6 分）

At least how many equilateral triangular cardboards of side length 1 are needed to form a convex pentagon, if overlapping of the cardboards is not allowed? (6 marks)

14. 現有一行 10 個抽屜，其中只有兩個相鄰的抽屜各有一本書，其餘抽屜則是空的。最少需要逐一打開多少個抽屜，才可保證能把兩本書都拿出來？（6 分）

There are 10 drawers in a row. Among them, only two consecutive drawers each contains a book, while the rest are empty. At least how many drawers should be opened one by one to ensure that we can take out both books? (6 marks)

15. 對任意整數 a 和 b ，定義 $a \star b = ab + 2a + 2b$ 。現將數字 1、2、3、4 分別填入算式「 $[(\square \star \square) \star \square] \star \square$ 」的空格中（每個數字只能使用一次），求所得結果的最大可能值。（6 分）

For any integers a and b , define $a \star b = ab + 2a + 2b$. The numbers 1, 2, 3 and 4 are now filled into the squares of the equation ' $[(\square \star \square) \star \square] \star \square$ ' (each number can only be used once). Find the greatest possible value of the result. (6 marks)

16. 老師對陳同學和李同學說：「我的計算機上現時顯示了一個四位正整數 n ，它是 9 的倍數，且其千位是 2。稍後，我會讓陳同學知道 n 的百位和十位，並讓李同學知道 n 的十位和個位。你們猜猜 n 的值吧！」老師之後一如所述讓兩位同學知道 n 的其中兩個位，可是陳同學仍然無法推斷出 n 的值。知道陳同學未能猜出 n 後，李同學依然無法知道 n 的值。已知兩位同學都是聰明的（即當有足夠資料時可以作出正確的分析），求 n 的最大可能值。 (6 分)

The teacher told two students, Chan and Lee: 'My calculator is now displaying a four-digit positive integer n which is a multiple of 9 and whose thousands digit is 2. In a moment I am going to let Chan know the hundreds and tens digits of n , and let Lee know the tens and unit digits of n . Try to guess what n is!' The teacher then let each of the two students know two of the digits of n as said. Still, Chan was unable to deduce the value of n . Knowing that Chan failed, Lee was still unable to know what n is. Given that the students were clever (i.e. they could make the correct analysis whenever sufficient information is available), find the greatest possible value of n . (6 marks)

17. 求 $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{32}$ 的值，答案準確至最接近整數。 (6 分)

Find the value of $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{32}$ correct to the nearest integer. (6 marks)

18. 現有一個 5×5 的棋盤。小莊把其中 k 個方格塗上黑色，其餘的方格則塗上白色。媽媽和小莊在棋盤上進行以下遊戲：媽媽把一枚棋子放在棋盤左下角的方格，然後每次把棋子向上或向右移動一格，最後到達棋盤右上角的方格。每次移動棋子後，媽媽都會告訴小莊棋子當時位處的方格的顏色。事後小莊需要猜出棋子的移動路線。若小莊總能成功，求 k 的最小可能值。 (7 分)

There is a 5×5 chessboard. John colours k of the cells black and the rest of them white. He plays a game with his mother on the chessboard as follows. Mum puts a chess piece in the bottom left corner cell of the chessboard. She then moves the chess piece to the cell immediately above or immediately to the right, until reaching the upper right corner cell. Every time when the chess piece is moved, Mum will tell John the colour of the cell in which the chess piece lands. After the piece reaches the end, John will have to guess the route taken by the chess piece afterwards. If John can always succeed, find the smallest possible value of k . (7 marks)

19. 某班有 80 名學生，他們的學號分別為 1 至 80。老師把他們分成 k 組，當中 k 大於 1，使得每組的學生數目相同，且每組中所有學生的學號之和亦相同。求 k 的所有可能值之和。

(7 分)

In a class there are 80 students and they are assigned class numbers from 1 to 80. The teacher divided them into k groups, where k is greater than 1, each containing the same number of students. Furthermore, the sum of class numbers of the members is the same for each group. Find the sum of all possible values of k .

(7 marks)

20. 在學年開始時，小芬決定要在每次默書後記錄本學年截至該次默書的平均分。至今她已經默書 7 次，每次的分數均是 91 至 100 之間（包括首尾兩數）的不同整數。此外，她每次默書後錄得的平均分皆為整數。若她在第 7 次默書中得到 95 分，則她在第 6 次默書中得到多少分？

(7 分)

At the beginning of a school year, Fanny decided that after each dictation, she would record the average score of all the dictations she has had in the school year. By now she has had 7 dictations, and each time she obtained a different integer score between 91 and 100 (inclusive). Furthermore, the average scores she has recorded after each dictation are all integers. If she obtained 95 marks in the 7th dictation, how many marks did she get in the 6th dictation?

(7 marks)

全卷完

END OF PAPER