第十六屆培正數學邀請賽（**2017** 年）

16th Pui Ching Invitational Mathematics Competition (2017)

決賽（中三組）

Final Event (Secondary 3)

時限：**2** 小時

Time allowed: 2 hours

參賽者須知：

Instructions to Contestants:

(a) 本卷共設 20 題，總分為 100 分。

There are 20 questions in this paper and the total score is 100.

(b) 除特別指明外，本卷內的所有數均為十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

(c) 除特別指明外，所有答案須以數字的真確值表達，並化至最簡。不接受近似值。

Unless otherwise stated, all answers should be given in exact numerals in their simplest form. No approximation is accepted.

(d) 把所有答案填在答題紙指定的空位上。毋須呈交計算步驟。

Put your answers on the space provided on the answer sheet. You are not required to hand in your steps of working.

(e) 不得使用計算機。

The use of calculators is not allowed.

(f) 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

注意：決賽的規則與初賽不同。除特別指明外，所有答案須以數字的真確值表達，並化至最簡。不接受近似值。

Note: The rule in the Final Event is different from that in the Heat Event. Unless otherwise stated, all answers should be given in exact numerals in their simplest form. No approximation is accepted.

第 1 至第 4 題，每題 3 分。

Questions 1 to 4 each carries 3 marks.

1. 設 a、b、c 為合成數，p 為質數。若 $a+b+c=p$，求 p 的最小可能值。

   Let a, b, c be composite numbers and p be a prime number. If $a+b+c=p$, find the smallest possible value of p.

2. 若 $\sqrt{15+4\sqrt{14}}=\sqrt{a}+\sqrt{b}$，其中 a 和 b 為正整數，求 a^2+b^2 的值。

   If $\sqrt{15+4\sqrt{14}}=\sqrt{a}+\sqrt{b}$ where a and b are positive integers, find the value of a^2+b^2.

3. 某等比數列共有 10 項，其中首項是 2，公比是 4。若 10 項之積為 2^k，求 k 的值。

   A geometric sequence consists of 10 terms. The first term is 2 and the common ratio is 4. If the product of the 10 terms is 2^k, find the value of k.

4. 設 n 為正整數。若 n 可被 2、3、5 和 7 整除，但 n 的每位數字均不是 2、3、5 或 7，求 n 的最小可能值。

   Let n be a positive integer which is divisible by 2, 3, 5 and 7. If none of the digits of n is 2, 3, 5 or 7, find the smallest possible value of n.

第 5 至第 8 題，每題 4 分。

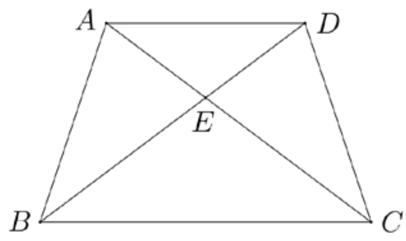
Questions 5 to 8 each carries 4 marks.

5. 設 n 為五位數。若把數字 2 放在 n 的首位之前，可組成六位數 a；若把數字 2 放在 n 的末位之後，可組成六位數 b。若 $b=3a$，求 n 的值。

   Let n be a five-digit number. A six-digit number a can be formed by appending the digit 2 to the front of n. A six-digit number b can be formed by appending the digit 2 to the end of n. If $b=3a$, find the value of n.

6. 圖中，$ABCD$ 是等腰梯形，其中 AD 與 BC 平行。E 是 AC 和 BD 的交點，且 $AE=3$、$BE=5$ 及 $\angle AEB=60^\circ$。求 $ABCD$ 的面積。

   In the figure, $ABCD$ is an isosceles trapezium in which AD is parallel to BC. E is the intersection of AC and BD with $AE=3$, $BE=5$ and $\angle AEB=60^\circ$. Find the area of $ABCD$.



7. 在數列 3, 6, 12, 14, 18, … 中，從第二項起每項均等於它的前一項加上其個位數（因此 18 之後的一項為 $18+8=26$）。求數列的第 2017 項。

   In the sequence 3, 6, 12, 14, 18, …, each term from the second term onwards is equal to the previous term added to its unit digit (hence the term after 18 is $18+8=26$). Find the 2017th term of the sequence.

8. 有多少種方法在一個 2×4 的表格的格子中分別填上 1 至 8 這 8 個整數，使得每列和每行中的整數均按左至右和上至下的次序遞增？

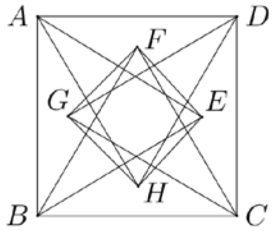
   In how many ways can the cells of a 2×4 table be filled with the 8 integers from 1 to 8 such that the numbers in each row and column follow ascending order, from left to right and from top to bottom?

第 9 至第 12 題，每題 5 分。

Questions 9 to 12 each carries 5 marks.

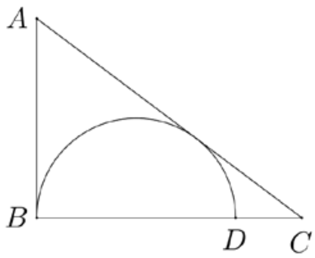
9. 圖中，$ABCD$ 是邊長為 4 的正方形。點 E、F、G 和 H 在正方形內，使得 ABE、BCF、CDG 和 DAH 均為等邊三角形。求 $EFGH$ 的面積。

   In the figure, $ABCD$ is a square of side length 4. Points E, F, G and H lie inside the square such that ΔABE, ΔBCF, ΔCDG and ΔDAH are all equilateral. Find the area of $EFGH$.



10. 圖中，ABC 是直角三角形，其中 B 是直角。D 是 BC 上的一點，使得 $BD = 2CD = 10$。若以 BD 為直徑的半圓與 AC 相切，求 AB 的長度。

    In the figure, ΔABC is right-angled at B. D is a point on BC such that $BD = 2CD = 10$. If the semi-circle with diameter BD is tangent to AC, find the length of AB.



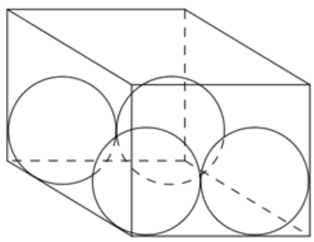
11. 在所示的算式中，每個字母代表一個由 0 至 9 的不同數字。已知 $\mathrm{A} < \mathrm{B} < \mathrm{C}$ 及 $\mathrm{A} < \mathrm{D} < \mathrm{E} < \mathrm{F}$，求 CAFE 所代表的四位數。

    In the addition shown, each letter represents a different digit from 0 to 9. Given that $\mathrm{A} < \mathrm{B} < \mathrm{C}$ and $\mathrm{A} < \mathrm{D} < \mathrm{E} < \mathrm{F}$, find the four-digit number represented by CAFE.

		A	B	C
+		D	E	F
	G	H	I	J

12. 現有一個長方盒子，裏面裝有四個半徑為 5 的球體，使得每個圓球體與盒子的其中三面相切，並與兩個相鄰的球體相切。若另一個半徑同樣為 5 的球體被放在這四個球體之上，使其與四個球體均相切，則這個球體的中心距離盒子底部的高度是多少？

    There is a rectangular box containing four spheres with radii 5, such that each sphere touches three faces of the box and touches two neighbouring spheres. If another sphere of radius 5 is put on top of the four spheres so that it is tangent to all four spheres, what will be the height of the centre of this sphere from the base of the box?

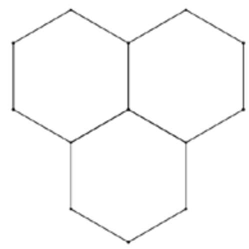


第 13 至第 16 題，每題 6 分。

Questions 13 to 16 each carries 6 marks.

13. 現有一些長度相同的木條。如圖所示，我們可利用 15 支木條組成 3 個互不重疊的正六邊形。若要在平面上組成 10 個互不重疊的正六邊形，最少需要多少支木條？

    There are some sticks of the same length. As shown in the figure, one can use 15 sticks to form 3 non-overlapping regular hexagons. What is the least number of sticks needed to form 10 non-overlapping regular hexagons on a plane?



14. 一個電腦遊戲中共有四個按鈕，分別會使遊戲中的主角向上、下、左或右移動 1 個單位。先後四次隨機按下其中一個按鈕後，主角距離起點 x 個單位。求 x 的期望值。

    There are four buttons in a computer game, which will move the character by 1 unit upward, downward, to the left and to the right respectively. In each round, a randomly-chosen button is pressed. After 4 rounds, the character ends at a distance of x units from the starting point. Find the expected value of x.

15. 某乒乓球比賽有 n 人參加，每名參賽者均與其餘各人對賽一次，而且沒有和局。小欣打算比賽結束後向其中三名參賽者索取簽名，當中其中一人必須在比賽中擊敗另外兩人。小欣發現，不論賽果如何，選出這三名參賽者均有最少 10395 種方法。求 n 的最小可能值。

    There are n participants in a table tennis tournament. Each participant plays one game against every other participant, and there is no tie in each game. Yanni plans to obtain the autographs of three players after the tournament, with the condition that one of them must have beaten the other two during the tournament. Yanni found that, regardless of the outcome of the games, she has at least 10395 ways to choose these three players. Find the smallest possible value of n.

16. 現要在算式「1□2□⋯□9」中的 8 個空格內都分別填入「+」、「−」或「×」，使其結果是一個偶數。共有多少種不同的方法填入符號？

    Each of the 8 squares in the expression ‘1□2□⋯□9’ is to be filled with one of the symbols ‘+’, ‘−’ or ‘×’ so that the result is an even number. In how many different ways can this be done?

第 17 至第 20 題，每題 7 分。

Questions 17 to 20 each carries 7 marks.

17. 設 m、n 為正整數，使得 $125(m^2+n^2)=(m+n)^3$。求 m 的所有可能值之和。

    Let m, n be positive integers such that $125(m^2+n^2)=(m+n)^3$. Find the sum of all possible values of m.

18. 考慮 $x+y=-100$、$x+y=-99$、…、$x+y=100$；$x-y=-100$、$x-y=-99$、…、$x-y=100$ 及 $x=-100$、$x=-99$、…、$x=100$ 共 603 條直線。它們共構成多少個交點？

    Consider the 603 straight lines $x+y=-100$, $x+y=-99$, …, $x+y=100$; $x-y=-100$, $x-y=-99$, …, $x-y=100$ and $x=-100$, $x=-99$, …, $x=100$. How many points of intersection do they form?

19. 設 a、b、c、d 為非零整數，使得 $3a^2+20ac+31c^2=-1$ 及 $3ab+10bc+10ad+31cd=0$。求 $3b^2+20bd+31d^2$ 的最小可能值。

    Let a, b, c, d be nonzero integers such that $3a^2+20ac+31c^2=-1$ and $3ab+10bc+10ad+31cd=0$. Find the smallest possible value of $3b^2+20bd+31d^2$.

20. 桌子上有 4 張正面朝下的咭片，現每次隨機選一張咭片並將其翻轉，直至所有咭片均是正面朝上為止。以 n 表示把咭片翻轉的總次數。求 n 是 4 的倍數的概率。

    There are 4 cards on the table, initially all facing downwards. Each time one of the cards is randomly chosen and flipped, until all cards are facing upwards. Let n denote the total number of flippings. Find the probability that n is a multiple of 4.

全卷完

END OF PAPER