

第十六屆培正數學邀請賽（2017 年）

16th Pui Ching Invitational Mathematics Competition (2017)

決賽（中一組）

Final Event (Secondary 1)

時限：2 小時

Time allowed: 2 hours

參賽者須知：

Instructions to Contestants:

- (a) 本卷共設 20 題，總分為 100 分。

There are 20 questions in this paper and the total score is 100.

- (b) 除特別指明外，本卷內的所有數均為十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

- (c) 除特別指明外，所有答案須以數字的真確值表達，並化至最簡。不接受近似值。

Unless otherwise stated, all answers should be given in exact numerals in their simplest form.
No approximation is accepted.

- (d) 把所有答案填在答題紙指定的空位上。毋須呈交計算步驟。

Put your answers on the space provided on the answer sheet. You are not required to hand in your steps of working.

- (e) 不得使用計算機。

The use of calculators is not allowed.

- (f) 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

注意：決賽的規則與初賽不同。除特別指明外，所有答案須以數字의真確值表達，並化至最簡。不接受近似值。

Note: The rule in the Final Event is different from that in the Heat Event. Unless otherwise stated, all answers should be given in exact numerals in their simplest form. No approximation is accepted.

第 1 至第 4 題，每題 3 分。

Questions 1 to 4 each carries 3 marks.

1. 設 a 、 b 、 c 為合成數， p 為質數。若 $a+b+c=p$ ，求 p 的最小可能值。

Let a, b, c be composite numbers and p be a prime number. If $a+b+c=p$, find the smallest possible value of p .

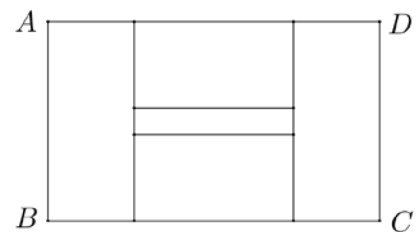
2. 若正整數 a 、 b 和 c 滿足 $\frac{1}{a}:\frac{2}{b}:\frac{3}{c}=4:5:6$ ，求 $a+b+c$ 的最小可能值。

If positive integers a, b and c satisfy $\frac{1}{a}:\frac{2}{b}:\frac{3}{c}=4:5:6$, find the smallest possible value of $a+b+c$.

3. 某圖書館逢星期日及在「日」是 5 的倍數的日子均會休館。在某個月中，該圖書館最多會休館多少天？

A library is closed every Sunday and whenever the 'day' in the date is a multiple of 5. What is the maximum number of days the library closes in a certain month?

4. 如圖所示，長方形 $ABCD$ 由 5 個小長方形組成，其中外面的 4 個小長方形全等，而中間的小長方形面積則是其中一個外面的小長方形面積的三分之一。求 $\frac{AB}{AD}$ 的值。



In the figure, the rectangle $ABCD$ is formed by 5 smaller rectangles, where the 4 outer rectangles are congruent and the inner rectangle has area equal to one-third that of one of the outer rectangles. Find the value of $\frac{AB}{AD}$.

第 5 至第 8 題，每題 4 分。

Questions 5 to 8 each carries 4 marks.

5. 設 n 為五位數。若把數字 2 放在 n 的首位之前，可組成六位數 a ；若把數字 2 放在 n 的末位之後，可組成六位數 b 。若 $b = 3a$ ，求 n 的值。

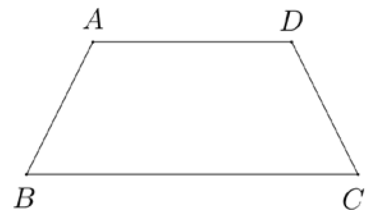
Let n be a five-digit number. A six-digit number a can be formed by appending the digit 2 to the front of n . A six-digit number b can be formed by appending the digit 2 to the end of n . If $b = 3a$, find the value of n .

6. 在數列 1, 2, 5, 29, ... 中，從第三項起每項均等於前兩項的平方和（因此 29 之後的一項為 $5^2 + 29^2 = 866$ ）。求數列第 2017 項的個位數字。

In the sequence 1, 2, 5, 29, ..., each term starting from the third term onwards is equal to the sum of squares of the previous two terms (hence the term after 29 is $5^2 + 29^2 = 866$). Find the unit digit of the 2017th term.

7. 設 $ABCD$ 為等腰梯形，其中 AD 與 BC 平行。若 $AB = CD = 4$ 、 $AD = 6$ ，且 $\angle A = \angle D = 135^\circ$ ，求 $ABCD$ 的面積。

Let $ABCD$ be an isosceles trapezium with AD parallel to BC . If $AB = CD = 4$, $AD = 6$ and $\angle A = \angle D = 135^\circ$, find the area of $ABCD$.



8. 設 m 、 n 為正整數，使得 $m^2 + 3n = 2017$ 。那麼， (m, n) 有多少組不同的可能值？

Let m, n be positive integers such that $m^2 + 3n = 2017$. How many different sets of possible values for (m, n) are there?

第 9 至第 12 題，每題 5 分。

Questions 9 to 12 each carries 5 marks.

9. 某直角三角形的斜邊長度為 50，其周長則為 110。求它的面積。

The hypotenuse of a right-angled triangle is 50 and its perimeter is 110. Find its area.

10. 一個電腦遊戲中共有四個按鈕，分別會使遊戲中的主角向上、下、左或右移動 1 個單位。有多少種不同的方法按四次按鈕，使主角回到起點？

There are four buttons in a computer game, which will move the character by 1 unit upward, downward, to the left and to the right respectively. How many ways are there to press the buttons four times so that the character returns to the starting point?

11. 現有兩隻電子錶。第一隻錶走 1 小時需比正常時間多用 3 分鐘，第二隻錶走 1 小時所需的時間則比正常少 3 分鐘。兩隻錶在每分鐘的開始時均會發出「嗶」聲。在正午時，兩隻錶被調整至正確時間。那麼，經過多少分鐘後，兩隻錶才會再同時發出「嗶」聲？

There are two digital watches. When the first watch has run 1 hour, the actual time is 3 minutes faster. When the second watch has run 1 hour, the actual time is 3 minutes slower. Both watches give a 'beep' sound at the beginning of every minute. At noon, the watches are adjusted to the correct time. How many minutes will pass before both watches give 'beep' sounds at the same time again?

12. 在所示的算式中，每個字母代表一個由 0 至 9 的不同數字。已知 $A < B < C$ 及 $A < D < E < F$ ，求 CAFE 所代表的四位數。

In the addition shown, each letter represents a different digit from 0 to 9. Given that $A < B < C$ and $A < D < E < F$, find the four-digit number represented by CAFE.

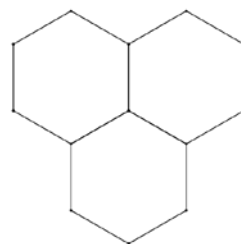
$$\begin{array}{r} A B C \\ + D E F \\ \hline G H I J \end{array}$$

第 13 至第 16 題，每題 6 分。

Questions 13 to 16 each carries 6 marks.

13. 現有一些長度相同的木條。如圖所示，我們可利用 15 支木條組成 3 個互不重疊的正六邊形。若要在平面上組成 10 個互不重疊的正六邊形，最少需要多少支木條？

There are some sticks of the same length. As shown in the figure, one can use 15 sticks to form 3 non-overlapping regular hexagons. What is the least number of sticks needed to form 10 non-overlapping regular hexagons on a plane?



14. 某乒乓球比賽有 2017 人參加，每名參賽者均與其餘各人對賽一次，而且沒有和局。小欣打算比賽結束後向其中三名參賽者索取簽名，當中其中一人必須在比賽中擊敗另外兩人。視乎賽果，這三名參賽者的可能組合數目有所不同。若此數目的最小可能值為 m ，求 m 除以 2015 時的餘數。

There are 2017 participants in a table tennis tournament. Each participant plays one game against every other participant, and there is no tie in each game. Yanni plans to obtain the autographs of three players after the tournament, with the condition that one of them must have beaten the other two during the tournament. Depending on the outcome of the games, the number of possible combinations of these three players varies. If the smallest possible value of this number is m , find the remainder when m is divided by 2015.

15. 方程 $(x-1)(x-3)\cdots(x-2017)=(x-2)(x-4)\cdots(x-2016)$ 有多少個實數解？

How many real number solutions are there to the following equation?

$$(x-1)(x-3)\cdots(x-2017)=(x-2)(x-4)\cdots(x-2016)$$

16. 在數列 $\frac{3}{3}, \frac{7}{5}, \frac{11}{7}, \frac{15}{9}, \dots$ 中，各項的分子組成一個公差為 4 的等差數列，各項的分母則組成一個公差為 2 的等差數列。老師要求每位同學在數列中選一項（可以相同），然後老師把每位同學所選的項乘起來，得到乘積 m 。若 m 是個三位整數，求 m 的最小可能值。

In the sequence $\frac{3}{3}, \frac{7}{5}, \frac{11}{7}, \frac{15}{9}, \dots$, the numerators of the terms form an arithmetic sequence with common difference 4, while the denominators of the terms form an arithmetic sequence with common difference 2. A teacher asks each student to pick a term from the sequence (possibly the same), and the teacher then multiplies the terms the students picked to obtain the product m . If m is a three-digit integer, find the smallest possible value of m .

第 17 至第 20 題，每題 7 分。

Questions 17 to 20 each carries 7 marks.

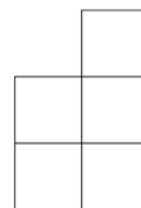
17. 設 m 、 n 為正整數，使得 $125(m^2 + n^2) = (m + n)^3$ 。求 m 的所有可能值之和。

Let m, n be positive integers such that $125(m^2 + n^2) = (m + n)^3$. Find the sum of all possible values of m .

18. 在直角座標平面上， x 座標和 y 座標皆為整數的點稱為「格點」。考慮 $x + y = -100$ 、 $x + y = -99$ 、 \dots 、 $x + y = 100$ ； $x - y = -100$ 、 $x - y = -99$ 、 \dots 、 $x - y = 100$ 及 $x = -100$ 、 $x = -99$ 、 \dots 、 $x = 100$ 共 603 條直線。它們所構成的交點中，有多少個位於「格點」？

On the Cartesian plane, points whose x - and y -coordinates are both integers are called 'lattice points'. Consider the 603 straight lines $x + y = -100$, $x + y = -99$, ..., $x + y = 100$; $x - y = -100$, $x - y = -99$, ..., $x - y = 100$ and $x = -100$, $x = -99$, ..., $x = 100$. Among the points of intersection formed by these lines, how many lie on a 'lattice point'?

19. 圖中的形狀由 5 個單位正方形組成。現於每個單位正方形填上一個不同的正整數，然後印出此形狀的 3 個複製本。這 3 個複製本之後用作覆蓋一個 4×4 的表格（允許將形狀旋轉和翻轉，但不能重疊），使得表格 16 個單位正方形的其中 15 個被覆蓋。覆蓋後發現表格內每橫行的各數之和均等於 S 。求 S 的最小可能值。



The figure is made up of 5 unit squares. A different positive integer is filled into each square, and then three copies of the figure are made. These three copies are then used to cover a 4×4 table (rotation and flipping of the copies are allowed while overlapping is not) so that 15 of the 16 unit squares of the table are covered. It turns out that the sum of the numbers on each horizontal row of the table is equal to S . Find the smallest possible value of S .

20. 某次遊戲中，兩班各派 9 名同學參加。這 18 人需分成 9 組，每組 2 人。在每班的 9 名學生中，其中 3 人只想與同班的同學一組，另有 3 人只想與另一班的同學一組，其餘 3 人則沒有任何要求。那麼，共有多少種不同的分組方法？

In a game, two classes each send 9 students to participate. These 18 students are to be divided into 9 groups of 2 students each. Among the 9 students of each class, 3 of them only want to pair up with someone from the same class, 3 of them only want to pair up with someone from the other class, while the other 3 have no preference. How many different possible groupings are there?

全卷完

END OF PAPER