第十五屆培正數學邀請賽（2016 年）

15th Pui Ching Invitational Mathematics Competition (2016)

初賽（中四組）

Heat Event (Secondary 4)

時限：1 小時 15 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

(a) 本卷共設 20 題，總分為 100 分。

There are 20 questions in this paper and the total score is 100.

(b) 除特別指明外，本卷內的所有數均為十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

(c) 作答時，每題的答案均須以 0 至 9999 之間的整數表示。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

Each answer must be given in the form of an integer between 0 and 9999. Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

(d) 不得使用計算機。

The use of calculators is not allowed.

(e) 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

注意：每題的答案均須以 0 至 9999 之間的整數表示，如有需要應以上述範圍內最接近正確答案的整數回答。如有兩個這樣的整數與正確答案同樣接近，則以「四捨五入」的原則取較大的整數。請細閱答題紙上的指示。

Note: Each answer must be given in the form of an integer between 0 and 9999. Where necessary, the answer should be rounded off to the nearest integer in the above range. Read the instructions on the answer sheet in detail.

1. 有多少個兩位數的數字之積是 5 的倍數？（3 分）

How many two-digit numbers have their product of digits being a multiple of 5? (3 marks)

2. 在首 40 個正整數中，有多少個可寫成兩個或以上連續正整數之積？（3 分）

How many of the first 40 positive integers can be written as the product of two or more consecutive positive integers? (3 marks)

3. 設 m、n 為正整數。若 $m<2n-4$ 且 $n<2m-7$，求 mn 的最小可能值。（3 分）

Let m and n be positive integers. If $m<2n-4$ and $n<2m-7$, find the smallest possible value of mn. (3 marks)

4. 求最小的三位平方數，使得它除以 7 的餘數是 4。（3 分）

Find the smallest three-digit square number which leaves a remainder of 4 when divided by 7. (3 marks)

5. 敏怡在計算 $\frac{1}{2\times1}+\frac{1}{3\times2}+\cdots+\frac{1}{100\times99}$ 時，誤把其中一個「×」號看成「−」號，結果得出的答案與正確答案相差 d。若 d 的最大可能值以最簡分數表示時為 $\frac{a}{b}$，求 a 的值。（4 分）

When computing $\frac{1}{2\times1}+\frac{1}{3\times2}+\cdots+\frac{1}{100\times99}$, Mandy mistakenly read one of the '×' signs as a '−' sign. As a result, her answer differed from the correct answer by d. If the greatest possible value of d in lowest form is $\frac{a}{b}$, find the value of a. (4 marks)

6. 拋物線 $2x-3y-y^2=2016$ 的頂點的 x 座標是甚麼？ （4分）

What is the x-coordinate of the vertex of the parabola $2x-3y-y^2=2016$? (4 marks)

7. 在某個課程中，學生必須在測驗取得一次合格方能畢業。學生可以在下列兩個方案任擇其一：

- 應考簡易版本的測驗，當中 60% 的學生會合格，但學生只有一次應考機會。
- 應考進階版本的測驗，當中每次只有 20% 的學生合格，但學生可以應考最多三次。

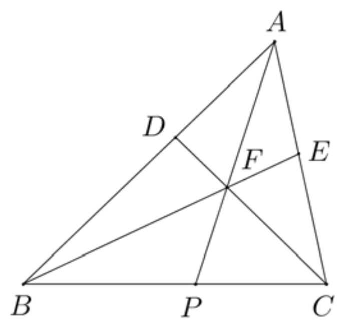
透過兩個方案成功畢業的學生比例相差多少個百份點？ （4分）

To complete a course, students must pass a test and they may choose one of the following two options:

- In the simple version of the test, 60% of the students can pass, but students are allowed only one attempt.
- In the advanced version of the test, only 20% of the students can pass each time. However, students are allowed up to three attempts.

What is the difference (in percentage points) between the proportions of students who can complete the course via the two options? (4 marks)

8. 在 ΔABC 中，$\angle BAC=48^\circ$ 及 $\angle ABC=56^\circ$。設 D 和 E 分別為邊 AB 和 AC 上的點，BE 和 CD 相交於 F，且 AF 的延長線交 BC 於 P。若 B、C、E 和 D 四點共圓；A、D、F 和 E 四點亦共圓，且 $\angle APB=x^\circ$，求 x 的值。 （4分）



In ΔABC, $\angle BAC=48^\circ$ and $\angle ABC=56^\circ$. Let D and E be points on sides AB and AC respectively, BE meets CD at F, and the extension of AF meets BC at P. If B, C, E, D are concyclic, A, D, F, E are concyclic, and $\angle APB=x^\circ$, find the value of x. (4 marks)

9. 在首 1000 個正整數中，有多少個以十六進制表示時的每位數字都介乎 0 至 9 之間？（在十六進制中，每位數字可以是 0 至 9 或 A 至 F，其中 A 至 F 分別代表 10 至 15。） （5 分）

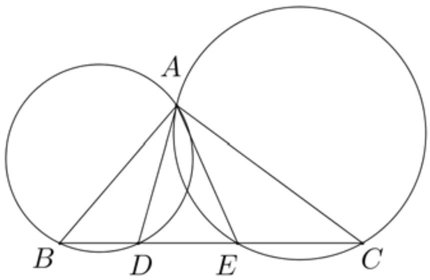
How many of the first 1000 positive integers have the property that each digit of their hexadecimal representation is between 0 and 9? (In hexadecimal representation, each digit can be from 0 to 9 or A to F, where A to F represent 10 to 15 respectively.) (5 marks)

10. 設 m、n 為正整數，其中 $m \geq n$。若它們的最小公倍數和最大公因數之差等於 $m+n$，求 $\frac{m}{n}$ 的值。 （5 分）

Let m and n be positive integers with $m \geq n$. If the difference between their L.C.M. and H.C.F. is equal to $m+n$, find the value of $\frac{m}{n}$. (5 marks)

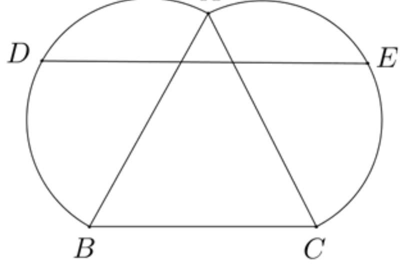
11. 圖中，D 和 E 是 ΔABC 的邊 BC 上的兩點，使得 $\angle BAD = \angle DAE = \angle EAC$。若 AD 是 ΔACE 外接圓的切線，AE 是 ΔABD 外接圓的切線，且 $\angle DAE = x°$，求 x 的值。 （5 分）

In the figure, D and E are two points on side BC of ΔABC such that $\angle BAD = \angle DAE = \angle EAC$. If AD is a tangent to the circumcircle of ΔACE, AE is a tangent to the circumcircle of ΔABD and $\angle DAE = x°$, find the value of x. (5 marks)



12. 圖中，ABC 是等邊三角形，邊長為 10。分別以 AB 和 AC 為直徑向三角形外作半圓，設 D 和 E 為兩個半圓上的點，使得 $AD = BD$ 及 $AE = CE$。求 DE 的長度。 （5 分）

In the figure, ABC is an equilateral triangle with side length 10. Construct semicircles outside the triangle with AB and AC as diameters respectively. Let D and E be points on the two semicircles such that $AD = BD$ and $AE = CE$. Find the length of DE. (5 marks)



13. 若 $y+4=(x-2)^2$ 及 $x+4=(y-2)^2$，且 $x\neq y$，求 x^2+y^2 的值。（6 分）

If $y+4=(x-2)^2$ and $x+4=(y-2)^2$ with $x\neq y$, find the value of x^2+y^2. (6 marks)

14. 在直角座標平面上，x 座標和 y 座標均為整數且 $x+y$ 為偶數的點稱為「偶格點」。那麼，以 $(0,0)$ 、$(39,41)$ 、$(80,-2)$ 和 $(41,-43)$ 為頂點的平行四邊形內（不包括邊界）有多少個「偶格點」？（6 分）

On the Cartesian plane, a point is called an 'even lattice point' if both its x-coordinate and y-coordinate are integers and $x+y$ is even. How many 'even lattice points' are there inside the parallelogram (excluding the boundary) with vertices $(0,0)$, $(39,41)$, $(80,-2)$ and $(41,-43)$? (6 marks)

15. 在所示的算式中，每個字母代表一個由 0 至 9 的不同數字。求 NINE 所代表的四位數的最大可能值。（6 分）

In the addition shown, each letter represents a different digit from 0 to 9. Find the greatest possible value of the four-digit number represented by NINE. (6 marks)

		O	N	E
		T	W	O
+		S	I	X
	N	I	N	E

16. 在直角座標平面上，所有滿足 $|2x-3y|\leq 12$ 及 $|2x+3y|\leq 12$ 的點 (x,y) 組成一個區域。求該區域的面積。（6 分）

On the Cartesian plane, all points (x,y) satisfying $|2x-3y|\leq 12$ and $|2x+3y|\leq 12$ form a region. Find the area of that region. (6 marks)

17. 已知首 m 個正奇數之和等於首 n 個正偶數之和加 212。求 n 的所有可能值之和。（7 分）

It is known that the sum of the first m odd positive integers is equal to the sum of the first n even positive integers plus 212. Find the sum of all possible values of n. (7 marks)

18. 已知不等式 $(1+n^{1^2-50})(1+n^{2^2-50})\cdots(1+n^{10^2-50})>1+n^k$ 對任意正整數 n 均成立。若 k 是正整數，求 k 的最大可能值。（7 分）

It is known that the inequality $(1+n^{1^2-50})(1+n^{2^2-50})\cdots(1+n^{10^2-50})>1+n^k$ holds for all positive integers n. If k is a positive integer, find the greatest possible value of k. (7 marks)

19. 黑板上寫有一行 4 個互不相同且不超過 100 的正整數。現於每兩個相鄰的數的下方寫下該兩個數的最大公因數加 1，從而獲得新的一行共 3 個正整數，然後重複相同步驟，如此類推，直至最後一行只有一個數為止。（下圖是一個例子，其中第二行首個數 7 是由 18 和 24 的最大公因數 6 再加上 1 得出的，其餘各項也相同。）求最後一行的那個數的最大可能值。（7 分）

18 24 7 49

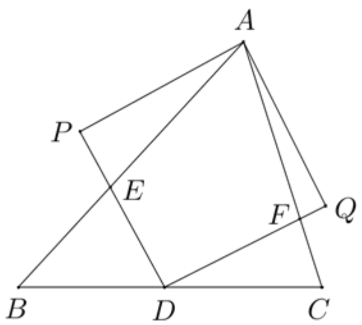
7 2 8

2 3

2

There is a row of 4 pairwise distinct positive integers not exceeding 100 written on the blackboard. Now below each pair of adjacent integers, we write down the H.C.F. of the two numbers plus 1. In this way we get a new row of 3 positive integers. The above procedure is repeated until the last row with just one number is obtained. (An example is illustrated above. The first number 7 in the second row is obtained by adding 1 to the H.C.F. of 18 and 24, i.e. 6, and same for the rest.) Find the greatest possible value of the number in the last row. (7 marks)

20. 在 ΔABC 中，D 是邊 BC 上的一點，使得 AD 平分 $\angle BAC$。現以 AD 為對角線構作正方形 $APDQ$，並設 AB 和 DP 交於 E，且 AC 和 DQ 交於 F。若 $\angle BAC = 60°$、$AB = 15$ 和 $AC = 10$，求四邊形 $AEDF$ 的面積。（7 分）



In ΔABC, D is a point on side BC such that AD bisects $\angle BAC$. Construct a square $APDQ$ using AD as a diagonal. Let AB and DP meet at E, and AC and DQ meet at F. If $\angle BAC = 60°$, $AB = 15$, $AC = 10$, find the area of the quadrilateral $AEDF$. (7 marks)

全卷完

END OF PAPER