

第十五屆培正數學邀請賽（2016 年）

15th Pui Ching Invitational Mathematics Competition (2016)

初賽（中一組）

Heat Event (Secondary 1)

時限：1 小時 15 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

- (a) 本卷共設 20 題，總分為 100 分。

There are 20 questions in this paper and the total score is 100.

- (b) 除特別指明外，本卷內的所有數均為十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

- (c) 作答時，每題的答案均須以 0 至 9999 之間的整數表示。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

Each answer must be given in the form of an integer between 0 and 9999. Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

- (d) 不得使用計算機。

The use of calculators is not allowed.

- (e) 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

注意：每題的答案均須以 0 至 9999 之間的整數表示，如有需要應以上述範圍內最接近正確答案的整數回答。如有兩個這樣的整數與正確答案同樣接近，則以「四捨五入」的原則取較大的整數。請細閱答題紙上的指示。

Note: Each answer must be given in the form of an integer between 0 and 9999. Where necessary, the answer should be rounded off to the nearest integer in the above range. Read the instructions on the answer sheet in detail.

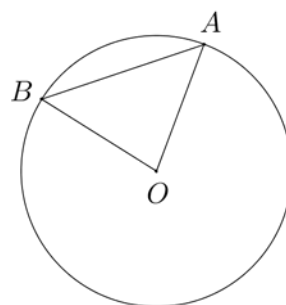
1. 有多少個兩位數的數字之和是 7? (3 分)  
How many two-digit numbers have sum of digits 7? (3 marks)

2. 求最小的質數，其中至少有兩個數字相同。 (3 分)  
Find the smallest prime number in which at least two of its digits are equal. (3 marks)

3. 求首 10 個個位數是 3 的正整數之和。 (3 分)  
Find the sum of the first 10 positive integers with unit digit 3. (3 marks)

4. 現有一個以  $O$  為中心、半徑為 10 的圓形。圓上有兩點  $A$  和  $B$ ，其中  $AB=10$ ，且  $\angle AOB = x^\circ$ 。求  $x$  的值。

There is a circle with centre  $O$  and radius 10. Points  $A$  and  $B$  are two points on the circle such that  $AB=10$  and  $\angle AOB = x^\circ$ . Find the value of  $x$ .



(3 分)

(3 marks)

5. 若在算式  $8\square4\square2\square1$  的方格中填上運算符號「+」、「-」、「 $\times$ 」或「 $\div$ 」（可重複使用），求所得出答案的最大可能值和最小可能值之差。 (4 分)

If the arithmetic operators '+', '-', ' $\times$ ' or ' $\div$ ' are inserted into the boxes of the expression  $8\square4\square2\square1$  (each operator can be used more than once), find the difference between the greatest possible value and the smallest possible value of the answer obtained.

(4 marks)

6. 有多少個兩位質數的十位和個位均為質數？ (4 分)

How many two-digit prime numbers are there in which both the unit digit and the tens digit are prime?

(4 marks)

7. 小欣在計算  $m-n$  時，誤把「 $-$ 」號看成「 $+$ 」號，結果得出的答案與正確答案相差了 2016。若  $n$  是正數，求  $n$  的值。 (4 分)

When computing  $m-n$ , Yannie mistakenly read the ' $-$ ' sign as a ' $+$ ' sign. As a result, her answer differed from the correct answer by 2016. If  $n$  is positive, find the value of  $n$ . (4 marks)

8. 一個圓形時鐘的秒針長  $x$  厘米。若秒針尖端每秒移動  $\frac{\pi}{2}$  厘米，求  $x$  的值。 (4 分)

The second hand of a circular clock has length  $x$  cm. If the tip of the second hand traverses  $\frac{\pi}{2}$  cm per second, find the value of  $x$ . (4 marks)

9. 設  $n$  是四位數。若把  $n$  的數字重新排列（允許以「0」開首），可得到不同的四位數。若其中兩個可得到的四位數為  $a$  和  $b$ ，求  $a-b$  的最大可能值。 (5 分)

Let  $n$  be a four-digit number. By rearranging the digits of  $n$  ('0' is allowed to be the leading digit), one can get different four-digit numbers. If  $a$  and  $b$  are two numbers that can be so obtained, find the greatest possible value of  $a-b$ . (5 marks)

10. 一張正方形咭紙沿某條直線被剪成兩份，兩部份分別是一個  $m$  邊形和一個  $n$  邊形，其中  $m \leq n$ 。那麼， $(m,n)$  有多少組不同的可能值？ (5 分)

A piece of square cardboard is cut into two parts along a straight line. The two parts are in the shape of an  $m$ -sided polygon and an  $n$ -sided polygon respectively, where  $m \leq n$ . How many different sets of possible values of  $(m,n)$  are there? (5 marks)

11. 現有兩行正整數。第一行的首項是 1，之後每項均等於該行前面所有數之和再加 1；第二行的首項是 1，之後每項均等於該行前面所有數之和再加 2。求兩行數的第 10 項之差。 (5 分)

There are two rows of positive integers. The first term of the first row is 1, and each subsequent term is the sum of all previous numbers in the same row plus 1. The first term of the second row is 1, and each subsequent term is the sum of all previous numbers in the same row plus 2. Find the difference between the 10th terms of the two rows. (5 marks)

12. 某兩位數等於它十位數字的立方與它個位數字的平方之和。求此數。 (5 分)

A two-digit number is equal to the sum of the cube of its tens digit and the square of its unit digit. Find this number. (5 marks)

13. 若  $a$ 、 $b$ 、 $c$ 、 $d$  四個數等於 2、4、6、8 (不一定按此順序)，求  $ab+bc+bd+cd$  的最大可能值。 (6 分)

If the four numbers  $a, b, c, d$  are equal to 2, 4, 6, 8 (not necessarily in this order), find the greatest possible value of  $ab+bc+bd+cd$ . (6 marks)

14. 五對夫婦參加一個遊戲，10 人當中有 4 人可以得獎，但規定夫妻二人不能同時得獎。那麼，得獎名單有多少個可能的組合？ (6 分)

Five couples participate in a game. 4 of the 10 participants can win prizes, subject to the condition that nobody can win a prize alongside his or her spouse. How many different possible combinations are there for the list of winners? (6 marks)

15. 在所示的算式中，每個字母代表一個由 0 至 9 的不同數字。求 NINE 所代表的四位數的最大可能值。 (6 分)

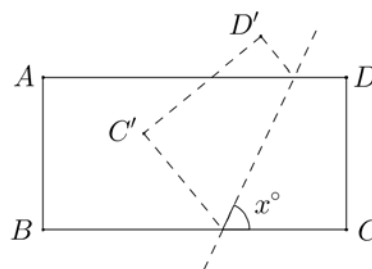
In the addition shown, each letter represents a different digit from 0 to 9. Find the greatest possible value of the four-digit number represented by NINE.

$$\begin{array}{r} \text{F I V E} \\ + \text{F O U R} \\ \hline \text{N I N E} \end{array}$$

(6 marks)

16. 如圖所示， $ABCD$  是一張長方形紙張，其中  $\angle CBD = 25^\circ$ 。若沿某條與邊  $BC$  所交的銳角為  $x^\circ$  的直線將紙張對摺，點  $C$  和  $D$  會分別被摺到點  $C'$  和  $D'$  上，且  $C'$  是  $B$  和  $D'$  的中點。求  $x$  的值。

In the figure,  $ABCD$  is a piece of rectangular paper, where  $\angle CBD = 25^\circ$ . If the paper is folded along a line which makes an acute angle of  $x^\circ$  with side  $BC$ , the points  $C$  and  $D$  will be folded to points  $C'$  and  $D'$  respectively, where  $C'$  is the midpoint of  $B$  and  $D'$ . Find the value of  $x$ .



(6 分)

(6 marks)

17. 有一個  $3 \times 3$  的方格表，其格子分別以 1 至 9 編號。現要把方格表的部分格子塗上陰影，並且規定每一行及每一列均最少有一格塗上陰影。那麼，塗上陰影的方法有多少種？ (7 分)

There is a  $3 \times 3$  grid, whose cells are numbered 1 to 9. Now some cells are to be shaded subject to the condition that at least one cell is shaded in every row and every column. How many different shadings are there? (7 marks)

18. 有多少種方法選取兩個不超過 100 的正整數，使得兩數的數字均沒有相同？（例如：可以選取 2 和 10，但不能選取 11 和 23，亦不能選取 15 和 59。） (7 分)

How many ways are there to choose two positive integers not exceeding 100, so that none of the digits among the two numbers are the same? (For example, we may choose 2 and 10, but not 11 and 23, nor 15 and 59.) (7 marks)

19. 現有一部特別的計算機，上面有一個紅色的按鈕。輸入一個正整數後按下該紅色按鈕，螢幕上會顯示原數的數字之和的兩倍（例如，輸入 123456 後按下紅色按鈕，可得  $(1+2+3+4+5+6) \times 2 = 42$ ，再按可得  $(4+2) \times 2 = 12$ ）。現輸入一個不超過 10000 的正整數，再連續按紅色按鈕 2016 次，得到的答案為  $n$ 。求  $n$  所有可能值之和。 (7 分)

There is a special calculator with a red button. If the red button is pressed after inputting a positive integer, the number on the screen will become twice the sum of digits of the original number. (For example, after inputting 123456, pressing the red button gives  $(1+2+3+4+5+6) \times 2 = 42$ , and pressing the red button again will give  $(4+2) \times 2 = 12$ .) Now a positive integer not exceeding 10000 is input, and then the red button is pressed 2016 times, giving a final answer  $n$ . Find the sum of all possible values of  $n$ . (7 marks)

20. 某數列的通項是  $\frac{10^{n-1}}{1 \times 2 \times \cdots \times n}$ 。若數列的第  $k$  項小於 1，求  $k$  的最小可能值。 (7 分)

The general term of a sequence is  $\frac{10^{n-1}}{1 \times 2 \times \cdots \times n}$ . If the  $k$ -th term of the sequence is less than 1, find the smallest possible value of  $k$ . (7 marks)

全卷完

END OF PAPER