第十四屆培正數學邀請賽（2015 年）

14th Pui Ching Invitational Mathematics Competition (2015)

初賽（中三組）

Heat Event (Secondary 3)

時限：1 小時 15 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

(a) 本卷共設 20 題，總分爲 100 分。

There are 20 questions in this paper and the total score is 100.

(b) 除特別指明外，本卷內的所有數均爲十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

(c) 作答時，每題的答案均須以 0 至 9999 之間的整數表示。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

Each answer must be given in the form of an integer between 0 and 9999. Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

(d) 不得使用計算機。

The use of calculators is not allowed.

(e) 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

注意：每題的答案均須以 0 至 9999 之間的整數表示，如有需要應以上述範圍內最接近正確答案的整數回答。如有兩個這樣的整數與正確答案同樣接近，則以「四捨五入」的原則取較大的整數。請細閱答題紙上的指示。

Note: Each answer must be given in the form of an integer between 0 and 9999. Where necessary, the answer should be rounded off to the nearest integer in the above range. Read the instructions on the answer sheet in detail.

1. 若 n 個質數之積等於 2015，求 n 的值。 （3 分）

If the product of n prime numbers is 2015, find the value of n. (3 marks)

2. 若 n 是三位奇數，且其數字之和不超過 16，求 n 的最大可能值。 （3 分）

If n is a three-digit odd number whose sum of digits does not exceed 16, find the maximum possible value of n. (3 marks)

3. 平面上有兩個正方形，它們的對應邊並不平行。那麼它們最多有多少個交點？ （3 分）

There are two squares on the plane and their corresponding sides are not parallel. At most how many points of intersection do they have? (3 marks)

4. 一顆紅色骰子的六個面上分別寫上數字 1、2、3、4、6、12，另一顆藍色骰子的六個面上分別寫上數字 2、3、4、5、6、8。現投擲這兩顆骰子，若得到的數字之積是 12，有多少種不同的可能性？ （3 分）

The numbers 1, 2, 3, 4, 6, 12 are written on the six faces of a red die, while the numbers 2, 3, 4, 5, 6, 8 are written on the six faces of another blue die. The two dice are now thrown. If the product of the numbers obtained is 12, how many different possibilities are there? (3 marks)

5. 現於一行 2015 個格子內各填入一個數。若每個格內的數不超過 3，且任意連續三格內的數之和不超過 7，求 2015 個格內所有數之和的最大可能值。 （4 分）

A number is put inside each cell of a row of 2015 cells. If the number in each cell does not exceed 3 and the sum of numbers in any 3 consecutive cells does not exceed 7, find the greatest possible value of the sum of numbers in all 2015 cells. (4 marks)

6. 若把所有十位是 1 的正整數從小至大排列，則 2015 是第幾項？ （4分）

If all positive integers with tens digit 1 are arranged in ascending order, what is the position of 2015? (4 marks)

7. 某三角形三條邊的長度爲 16、30 和 34。若它最大的一隻內角是 $x°$，求 x。 （4分）

A triangle has side lengths 16, 30 and 34. If its largest interior angle is $x°$, find x. (4 marks)

8. 現有很多堆金幣。已知第一堆有 3 枚金幣，之後每堆金幣的數目都是前一堆的兩倍多一枚。那麼第 10 堆有多少枚金幣？ （4分）

There are many piles of coins. It is known that the first pile consists of 3 coins. Each subsequent pile contains one more coin than twice the previous pile. How many coins are there in the 10th pile? (4 marks)

9. 有多少個四位正整數的數字之積爲 60？ （5分）

How many four-digit positive integers have product of digits 60? (5 marks)

10. 有多少個四位正整數可被 7 整除但不可被 13 整除？ （5分）

How many four-digit positive integers are divisible by 7 but not 13? (5 marks)

11. 在所示的算式中，每個字母代表一個由 0 至 9 的不同數字。求 ABCE 所代表的四位數。 （5分）

In the multiplication shown, each letter represents a different digit from 0 to 9. Find the four-digit number represented by ABCE. (5 marks)

$$\begin{array}{cccccc} & & & A & B & C \\ \times & & & & B & A \\ \hline & A & A & B & D & \\ & & & C & E & D \\ \hline & A & F & F & E & D \end{array}$$

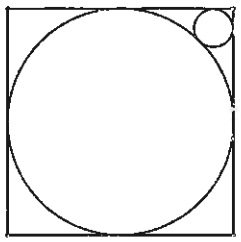
12. 某正整數除以 6、15 和 60 時，餘數分別是 2、8 和 n。求 n 的最大可能值。 （5分）

When a positive integer is divided by 6, 15 and 60, the remainders are 2, 8 and n respectively. Find the greatest possible value of n. (5 marks)

13. 設 a、b、c、d、e 爲正整數，其中 $2 \le a < b < c < d < e \le 100$。若這五個正整數依次組成一個等比數列，且 a 和 e 的最大公因數是 1，求 c 的值。（6分）

Let a, b, c, d, e be positive integers where $2 \le a < b < c < d < e \le 100$. If these five positive integers form a geometric sequence in this order, and that the H.C.F. of a and e is 1, find the value of c. (6 marks)

14. 圖中，大圓內接於正方形，小圓與大圓和正方形的兩邊相切。若大圓的半徑爲 60，求小圓的半徑。（6分）

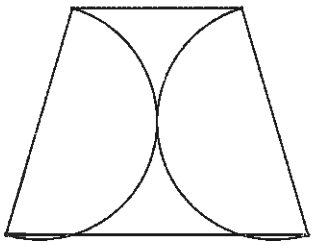


In the figure, the larger circle is inscribed in a square. The smaller circle is tangent to the larger circle and two sides of the square. If the larger circle has radius 60, find the radius of the smaller circle. (6 marks)

15. 某正方形的兩個對頂點的座標分別是 (0, 0) 和 (10, 10)。兩條穿過 (3, 4) 且分別平行於正方形兩條對角線的直線將正方形分成四部分。求最大部分的面積。（6分）

Two opposite vertices of a square have coordinates (0, 0) and (10, 10) respectively. Two lines which pass through (3, 4) and which are parallel to the two diagonals of the square respectively cut the square into four parts. Find the area of the largest part. (6 marks)

16. 如圖所示，一個等腰梯形的上底和下底的長度分別是 16 和 40。若分別以兩條斜邊爲直徑向梯形內作半圓，則該兩個半圓相切。求梯形的周界。（6分）



The figure shows an isosceles trapezium whose upper and lower bases have lengths 16 and 40 respectively. When semi-circles with the two legs as diameters are constructed towards the interior of the trapezium, the two semi-circles turn out to be tangent to each other. Find the perimeter of the trapezium. (6 marks)

17. 求 $\frac{99^3+100^3}{1^3+100^3}+\frac{97^3+99^3}{2^3+99^3}+\frac{95^3+98^3}{3^3+98^3}+\cdots+\frac{1^3+51^3}{50^3+51^3}$ 的值。 （7分）

Find the value of $\frac{99^3+100^3}{1^3+100^3}+\frac{97^3+99^3}{2^3+99^3}+\frac{95^3+98^3}{3^3+98^3}+\cdots+\frac{1^3+51^3}{50^3+51^3}$. (7 marks)

18. 一張遊戲咭上印有一個 4×4 方格表，參加者需把整數 1 至 16 分別填進方格表的 16 個格子，使得每個格子恰好有一個數。然後，主持人會從 1 至 16 隨機抽出 7 個幸運號碼。如果有其中 4 個幸運號碼位於同一行或同一列，參加者便可中獎。若中獎的概率以最簡分數表示時爲 $\frac{a}{b}$，求 $a+b$ 的值。 （7分）

A 4×4 table is printed on a game card. The participant has to fill in the numbers 1 to 16 into the 16 cells of the table such that there is exactly one number in each cell. The master of the game will then randomly draws 7 lucky numbers from 1 to 16. If 4 of the lucky numbers lie in the same row or the same column, the participant can win a prize. If the probability of winning a prize is $\frac{a}{b}$ in lowest form, find the value of $a+b$. (7 marks)

19. 若正整數 m 和 n 滿足 $\frac{23}{50}<\frac{m}{n}<\frac{64}{139}$，求 n 的最小可能值。 （7分）

If positive integers m and n satisfy $\frac{23}{50}<\frac{m}{n}<\frac{64}{139}$, find the smallest possible value of n. (7 marks)

20. 若一個正整數的平方的最後三位數字相同且非零，則該正整數稱爲「好數」，例如：因爲 $38^2=1444$，故 38 是「好數」。若把所有「好數」從小至大排列，求當中第二個數。 （7分）

If the last three digits of the square of a positive integer are the same and nonzero, we say that the positive integer is 'good'. For example, since $38^2=1444$, so 38 is 'good'. If we arrange all 'good' numbers in ascending order, what is the second number? (7 marks)

全卷完

END OF PAPER