第十四屆培正數學邀請賽（2015 年）
14th Pui Ching Invitational Mathematics Competition (2015)

初賽（中一組）
Heat Event (Secondary 1)

時限：1 小時 15 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

(a) 本卷共設 20 題，總分爲 100 分。

There are 20 questions in this paper and the total score is 100.

(b) 除特別指明外，本卷內的所有數均爲十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

(c) 作答時，每題的答案均須以 0 至 9999 之間的整數表示。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

Each answer must be given in the form of an integer between 0 and 9999. Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

(d) 不得使用計算機。

The use of calculators is not allowed.

(e) 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

1. 時鐘上的時針轉一圈需時多少分鐘？ （3分）

   How many minutes does it take for the hour hand of a clock to make one complete revolution? (3 marks)

2. 在一個 10×10 的方格表中，第二、三行和第二、三列合共有多少個格子？ （3分）

   In a 10×10 table, how many cells are there in total in the second and third rows together with the second and third columns? (3 marks)

3. 一顆紅色骰子的六個面上分別寫上數字 1、3、4、5、6、8，另一顆藍色骰子的六個面上分別寫上數字 0、2、4、6、8、10。現投擲這兩顆骰子，若得到的數字之和是 9，有多少種不同的可能性？ （3分）

   The numbers 1, 3, 4, 5, 6, 8 are written on the six faces of a red die, while the numbers 0, 2, 4, 6, 8, 10 are written on the six faces of another blue die. The two dice are now thrown. If the sum of the numbers obtained is 9, how many different possibilities are there? (3 marks)

4. 嘉琳每連續五天最多只會有一天到圖書館。那麼她在一月份中最多到了圖書館多少次？ （3分）

   Macy goes to the library at most once in any consecutive five days. At most how many times will Macy go to the library in January? (3 marks)

5. 一個四邊形的其中三隻內角分別是某三角形的三隻內角的 1.5 倍，而第四隻內角是 $x°$。求 x 的值。 （4分）

   Three interior angles of a quadrilateral are 1.5 times the three interior angles of a triangle respectively, while the fourth interior angle is $x°$. Find the value of x. (4 marks)

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6. 設 a、b、c 為正整數，使得 $a! \times b! \times c! = 25920$（其中 $m! = 1 \times 2 \times \cdots \times m$），求 a 的最大可能值。 （4分）

   Let a, b, c be positive integers such that $a! \times b! \times c! = 25920$ (where $m! = 1 \times 2 \times \cdots \times m$). Find the greatest possible value of a. (4 marks)

7. 一個等邊三角形的邊長與一個正六邊形相同。後者的面積是前者的多少倍？ （4分）

   An equilateral triangle has the same side length as a regular hexagon. How many times the area of the former is the area of the latter? (4 marks)

8. 求最小的四位正整數 n，它可被 9 整除，其四個數字均非零且互不相同。 （4分）

   Find the smallest four-digit positive integer n which is divisible by 9 and which consists of four pairwise distinct non-zero digits. (4 marks)

9. 現將 2、3、12、14、15、20、21 分成兩組，使得每組各數之積相等。求此積。 （5分）

   The numbers 2, 3, 12, 14, 15, 20, 21 are divided into two groups such that the product of the numbers in each group is the same. Find this product. (5 marks)

10. 已知 p、q、r 為質數。若 $pq + pr = 80$ 而 $pq + qr = 425$，求 $p + q + r$ 的值。 （5分）

    Let p, q, r be prime numbers. If $pq + pr = 80$ and $pq + qr = 425$, find the value of $p + q + r$. (5 marks)

11. 一個凸 n 邊形的內角（以「度」為單位時）均為正整數，且組成一個等差數列。求 n 的最大可能值。 （5分）

    The interior angles of a convex n-gon (in degrees) are all positive integers, and they form an arithmetic sequence. Find the greatest possible value of n. (5 marks)

12. 求所有小於 100 且可被 4 或 5 整除的正整數之和。 （5分）

    Find the sum of all positive integers smaller than 100 which are divisible by 4 or 5. (5 marks)

13. 有多少個四位正整數的四個數字依次組成一個等差數列（例如：1357、5678）？ （6分）

How many four-digit positive integers are there such that the four digits form an arithmetic sequence in that order (e.g. 1357, 5678)? (6 marks)

14. 在數列 $\{a_n\}$ 中，對任意不超過 2014 的正整數 n 皆有 $a_n + 2a_{2015-n} = 3n$。求 a_{10} 的值。 （6分）

In the sequence $\{a_n\}$, we have $a_n + 2a_{2015-n} = 3n$ for all positive integers n not exceeding 2014. Find the value of a_{10}. (6 marks)

15. 某次數學測驗的其中一道題如下：「寫出一個有一項或以上的數列，其每項均是 1、2、3、4、5 或 6，且沒有兩項相同。」這題有多少個不同的正確答案？ （6分）

A question in a mathematics test reads: 'Write down a sequence containing one or more terms such that each term is 1, 2, 3, 4, 5 or 6 and no two terms are the same'. How many different correct answers to this question are there? (6 marks)

16. 現將一個正方體分割成兩個長方體，使得它們的表面積比等於 3:2，體積比以最簡形式表示時為 $a:b$。求 $a+b$ 的值。 （6分）

A cube is cut into two cuboids such that the ratio of their surface areas is $3:2$, and the ratio of their volumes is $a:b$ in lowest form. Find the value of $a+b$. (6 marks)

17. 設 a、b、c、d 為正整數，使得 $a+b+c+d = 63$。求 $ab+bc+cd$ 的最大值。 （7分）

Let a, b, c, d be positive integers such that $a+b+c+d = 63$. Find the maximum value of $ab+bc+cd$. (7 marks)

18. 一個半徑為 3、夾角為 $x°$ 的扇形面積和周界的數值相等。求 x 的值。 （7分）

The area and perimeter of a sector with radius 3 and angle $x°$ are numerically equal. Find the value of x. (7 marks)

19. 設 n 為兩位正整數，其兩位數字均非零。若把 n 的兩位數字對調，可得正整數 m。已知 m 和 n 的最大公因數大於 5。那麼，n 有多少個不同的可能值？ （7分）

Let n be a two-digit positive integer whose both digits are nonzero. By switching the two digits of n, one obtains the positive integer m. Given that the H.C.F. of m and n is greater than 5, how many different possible values of n are there? (7 marks)

20. 如果某正整數由左至右和由右至左看皆相同，我們稱這個數為「回文數」。例如：3883、12321 和 25052 都是「回文數」。有多少種方法選兩個三位「回文數」，使得它們之和是一個四位「回文數」（不考慮次序，例如 (202, 909) 和 (909, 202) 視為相同選擇）？ （7分）

If a positive integer reads the same from left to right as from right to left, it is called a 'palindrome'. For example, 3883, 12321 and 25052 are 'palindromes'. How many ways are there to choose two three-digit 'palindromes' such that their sum is a four-digit 'palindrome'? (The order need not be taken into account. For example, (202, 909) and (909, 202) are regarded to be the same choice.) (7 marks)

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END OF PAPER

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