第十三屆培正數學邀請賽（**2014** 年）

13th Pui Ching Invitational Mathematics Competition (2014)

初賽（中二組）

Heat Event (Secondary 2)

時限：**1** 小時 **15** 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

(a) 本卷共設 20 題，總分為 100 分。

There are 20 questions in this paper and the total score is 100.

(b) 除特別指明外，本卷內的所有數均為十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

(c) 作答時，每題的答案均須以 0 至 9999 之間的整數表示。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

Each answer must be given in the form of an integer between 0 and 9999. Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

(d) 不得使用計算機。

The use of calculators is not allowed.

(e) 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

注意：每題的答案均須以 0 至 9999 之間的整數表示，如有需要應以上述範圍內最接近正確答案的整數回答。如有兩個這樣的整數與正確答案同樣接近，則以「四捨五入」的原則取較大的整數。請細閱答題紙上的指示。

Note: Each answer must be given in the form of an integer between 0 and 9999. Where necessary, the answer should be rounded off to the nearest integer in the above range. Read the instructions on the answer sheet in detail.

1. 已知在 1990 年時，語晴、曉嵐和穎琪三人的年齡之和爲 50。那麼在 2014 年時，她們三人的年齡之和是多少？（年齡以當年的年份減去其出生年份計算。） （3 分）

It is known that in 1990, the sum of the ages of Athena, Chloe and Vicky is 50. What is the sum of their ages in 2014? (The age is obtained by subtracting the year of birth from the current year). (3 marks)

2. 有多少個兩位正奇數？ （3 分）

How many two-digit positive odd numbers are there? (3 marks)

3. 已知 n 爲四位平方數，其十位爲奇數。求 n 的最小可能值。 （3 分）

Suppose n is a four-digit square number whose tens digit is odd. Find the smallest possible value of n. (3 marks)

4. 求最小的四位正整數 n，使得 n 的每個數字均非零，而且都不是 n 的因數。 （3 分）

Find the smallest four-digit positive integer n such that each digit of n is non-zero and is not a factor of n. (3 marks)

5. 如果某正整數由左至右和由右至左看皆相同，我們稱這個數爲「回文數」，例如：3883、12321 和 25052 都是「回文數」。若 n 是正整數而 $2014+n$ 是「回文數」，求 n 的最小可能值。 （3 分）

If a positive integer reads the same from left to right as from right to left, it is called a ‘palindrome’. For example, 3883, 12321 and 25052 are ‘palindromes’. If n is a positive integer while $2014+n$ is a ‘palindrome’, find the smallest possible value of n. (3 marks)

6. 已知兩個質數 p 和 q 之和為 2014。求 $\frac{p}{q}$ 的最大可能值。（4分）

Suppose the sum of two prime numbers p and q is 2014. Find the largest possible value of $\frac{p}{q}$. (4 marks)

7. 最多可從一個以 O 為圓心的圓上選取多少點，使得對任意兩個不同的被選取的點 A 和 B，均有 $\angle AOB > 25°$？（4分）

At most how many points can be chosen on a circle with centre O, such that $\angle AOB > 25°$ for any two distinct chosen points A and B? (4 marks)

8. 一個袋子裏裝有五個球，每個球上各寫有一個整數。現在晉生每次從袋子中拿出兩個球，並記下所得的兩個整數之和，然後把球重新放進袋子內。重覆這操作 10 次後，晉生所記下的整數分別為 110、112、113、114、115、116、117、118、120 和 121。求五個球上所寫的整數中最大的一個。（5分）

In a bag there are five balls, on each of which an integer is written. Anson takes two balls from the bag each time and notes down the sum of the two integers, and then puts the balls back to the bag. After repeating this for 10 times, the integers Anson obtained are 110, 112, 113, 114, 115, 116, 117, 118, 120 and 121. Find the largest number among the integers written on the five balls. (5 marks)

9. 設 n 為三位正整數。若 n^2 的最後三位數字從左至右為 369，求 n 的最小可能值。（5分）

Let n be a three-digit positive integer. If the last three digits of n^2 (from left to right) are 369, find the smallest possible value of n. (5 marks)

10. 已知 m、n 為兩位正整數，其中把 m 的兩位數字左右互換可得到 n。若 $m+n$ 和 $m-n$ 均為平方數，求 n 的值。（5分）

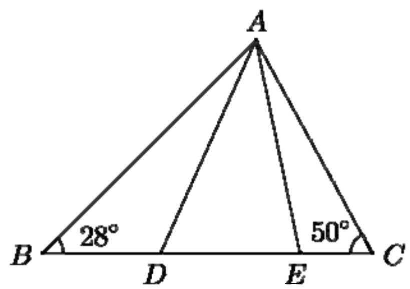
Let m and n be two-digit positive integers, such that n can be obtained by swapping the two digits of m. If $m+n$ and $m-n$ are both square numbers, find the value of n. (5 marks)

11. 最多可從一個正方形的四個頂點、四邊的中點和正方形中心這九點中選出多少個不同的點，使得當中任意四點均不構成一個矩形（包括任意方向的長方形和正方形）？ （5分）

At most how many points can be chosen from the following nine points: the four vertices of a square, the mid-points of its four sides and its centre, such that for any four chosen points, they do not form a rectangle (including rectangles and squares in any direction)? (5 marks)

12. 在 ΔABC 中，$\angle ABC = 28°$ 和 $\angle ACB = 50°$。D 和 E 為 BC 上的點，使得 $AB = BE$ 和 $AC = CD$。若 $\angle DAE = x°$，求 x 的值。 （5分）

In ΔABC, $\angle ABC = 28°$ and $\angle ACB = 50°$. D and E are points on BC such that $AB = BE$ and $AC = CD$. If $\angle DAE = x°$, find the value of x. (5 marks)



13. 現要安排四名男孩和四名女孩圍着一張圓桌就坐。若任意兩名男孩都不可相鄰而坐，則安排座位的方法有多少種？（如果每人左方的人在兩種方法中皆相同，則這兩種安排座位的方法視為相同。） （6分）

Four boys and four girls are to be seated at a round table and no two boys may sit next to each other. How many different seating arrangements are there? (Two seating arrangements are regarded to be the same if every person finds the same left-hand neighbour in the two arrangements.) (6 marks)

14. 設 $[x]$ 代表不超過 x 的最大整數，例如 $[2.1]=2$，$[4]=4$ 和 $[5.7]=5$。若 n 是小於 2014 的正整數，使得 $\left[\frac{3n}{2014}\right] < 2\left[\frac{2n}{2014}\right]$，則 n 有多少個不同的可能值？ （6分）

Let $[x]$ denote the largest integer not exceeding x. For example, $[2.1]=2$, $[4]=4$ and $[5.7]=5$. If n is an integer smaller than 2014 such that $\left[\frac{3n}{2014}\right] < 2\left[\frac{2n}{2014}\right]$, how many different possible values of n are there? (6 marks)

15. 若一個底半徑為 r 和高為 h 的圓柱體的體積和表面積分別為 12 和 216，求 $\frac{1}{r}+\frac{1}{h}$ 的值。（6分）

If a cylinder with base radius r and height h has volume 12 and surface area 216, find the value of $\frac{1}{r}+\frac{1}{h}$. (6 marks)

16. 若 $\frac{12}{2013}<\frac{m}{n}<\frac{13}{2014}$，其中 m 和 n 為正整數，求 n 的最小可能值。（6分）

If $\frac{12}{2013}<\frac{m}{n}<\frac{13}{2014}$, where m and n are positive integers, find the smallest possible value of n. (6 marks)

17. 設 $n=1+11+111+\cdots+11\ldots1$（其中最後一項有 2014 位）。那麼 n 的數字中有多少個是「1」？（7分）

Let $n=1+11+111+\cdots+11\ldots1$ (where the last term contains 2014 digits). How many digits of n are '1'? (7 marks)

18. 在所示的算式中，每個字母代表一個由 1 至 9 的不同數字。求 GHI 所代表的三位數的最小可能值。（7分）

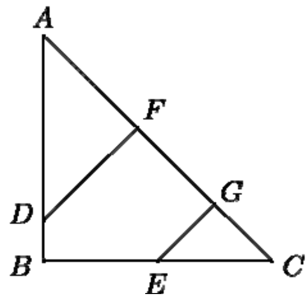
In the addition shown, each letter represents a different digit from 1 to 9. Find the smallest possible value of the three-digit number represented by GHI. (7 marks)

$$\begin{array}{cccc} & A & B & C \\ + & D & E & F \\ \hline & G & H & I \end{array}$$

19. 現要把整數 1 至 9 分別填入一個 3×3 的方格表的 9 格中，使得每行和每列的三個數之和均為奇數。若中間方格的數為 1，那麼有多少種方法填入餘下各數？（7分）

The integers from 1 to 9 are to be filled into the 9 cells of a 3×3 table, in a way such that the sum of the three numbers in each row and each column is an odd number. If the number in the central cell is 1, how many ways are there to fill in the remaining numbers? (7 marks)

20. 圖中，ABC 是等腰直角三角形，其中 $\angle ABC = 90°$ 且 $AB = BC = 36$。D 和 E 分別為 AB 和 BC 上的點，F 和 G 則為 CA 上的兩點，使得 DF 和 EG 均與 AC 垂直，且 $FG = 12\sqrt{2}$。求五邊形 $BEGFD$ 面積的最大可能值。（7分）



In the figure, ΔABC is right-angled and isosceles with $\angle ABC = 90°$ and $AB = BC = 36$. D and E are points on AB and BC respectively, while F and G are two points on CA such that both DF and EG are perpendicular to AC, and $FG = 12\sqrt{2}$. Find the greatest possible value of the area of pentagon $BEGFD$. (7 marks)

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END OF PAPER