第十三屆培正數學邀請賽（2014 年）

13th Pui Ching Invitational Mathematics Competition (2014)

初賽（中一組）

Heat Event (Secondary 1)

時限：1 小時 15 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

(a) 本卷共設 20 題，總分爲 100 分。

There are 20 questions in this paper and the total score is 100.

(b) 除特別指明外，本卷內的所有數均爲十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

(c) 作答時，每題的答案均須以 0 至 9999 之間的整數表示。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

Each answer must be given in the form of an integer between 0 and 9999. Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

(d) 不得使用計算機。

The use of calculators is not allowed.

(e) 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

注意：每題的答案均須以 0 至 9999 之間的整數表示，如有需要應以上述範圍內最接近正確答案的整數回答。如有兩個這樣的整數與正確答案同樣接近，則以「四捨五入」的原則取較大的整數。請細閱答題紙上的指示。

Note: Each answer must be given in the form of an integer between 0 and 9999. Where necessary, the answer should be rounded off to the nearest integer in the above range. Read the instructions on the answer sheet in detail.

1. 已知在 1990 年時，語晴、曉嵐和穎琪三人的年齡之和爲 50。那麼在 2014 年時，她們三人的年齡之和是多少？（年齡以當年的年份減去其出生年份計算。） （3 分）

   It is known that in 1990, the sum of the ages of Athena, Chloe and Vicky is 50. What is the sum of their ages in 2014? (The age is obtained by subtracting the year of birth from the current year). (3 marks)

2. 求 1227^3 除以 100 時的餘數。 （3 分）

   Find the remainder when 1227^3 is divided by 100. (3 marks)

3. 已知 n 爲四位平方數，其十位爲奇數。求 n 的最小可能值。 （3 分）

   Suppose n is a four-digit square number whose tens digit is odd. Find the smallest possible value of n. (3 marks)

4. 一個三角形的三隻內角分別爲 $x°$、$y°$ 和 $z°$。若 $2y = x + z$，求 y 的值。 （3 分）

   The three interior angles of a triangle are $x°$, $y°$ and $z°$ respectively. If $2y = x + z$, find the value of y. (3 marks)

5. 兩個正整數之和爲 73。求它們最大公因數的最大可能值。 （4 分）

   The sum of two positive integers is 73. Find the greatest possible value of their H.C.F. (4 marks)

6. 若某圓形的面積和周界的數值之比爲 $9:1$，求該圓形的半徑。 （4 分）

   If the ratio of numerical values of the area and perimeter of a circle is $9:1$, find the radius of the circle. (4 marks)

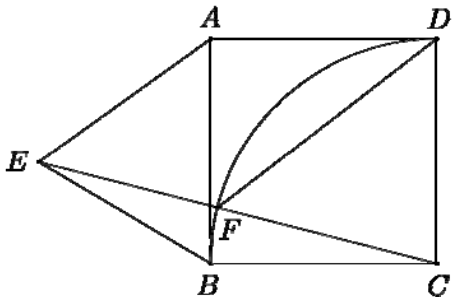
7. 已知兩個質數 p 和 q 之和爲 2014。求 $\frac{p}{q}$ 的最大可能值。 （4分）

Suppose the sum of two prime numbers p and q is 2014. Find the largest possible value of $\frac{p}{q}$. (4 marks)

8. 已知 n 爲兩位正整數，其數字之積爲 m。求 m 與 n 之差的最大可能值。 （4分）

Given that n is a two-digit positive integer with product of digits m, find the greatest possible value of the difference between m and n. (4 marks)

9. 圖中，$ABCD$ 爲正方形，E 是正方形外一點，使得 AEB 爲等邊三角形。以 C 爲中心、過點 B 和 D 的圓弧交線段 CE 於點 F。若 $\angle DFC = x°$，求 x 的值。 （5分）



In the figure, $ABCD$ is a square. E is a point outside the square such that ΔAEB is equilateral. The circular arc with centre C and passing through B and D meets the line segment CE at F. If $\angle DFC = x°$, find the value of x. (5 marks)

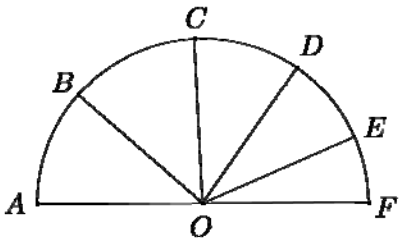
10. 嘉琳在一張長方形咭紙上畫上 6 條平行於邊界的直線，然後沿這些直線把咭紙剪開。那麼嘉琳最多可得到多少片咭紙？ （5分）

Macy drew 6 straight lines on a piece of rectangular cardboard such that each line is parallel to the boundary of the cardboard. Afterwards, she cut the cardboard along these lines. What is the maximum number of pieces of cardboards that Macy can get? (5 marks)

11. 最多可從一個正方形的四個頂點、四邊的中點和正方形中心這九點中選出多少個不同的點，使得當中任意四點均不構成一個矩形（包括任意方向的長方形和正方形）？ （5分）

At most how many points can be chosen from the following nine points: the four vertices of a square, the mid-points of its four sides and its centre, such that for any four chosen points, they do not form a rectangle (including rectangles and squares in any direction)? (5 marks)

12. 圖中，O 爲半圓的圓心，A、B、C、D、E、F 依次爲圓周上的點，而 AF 爲直徑。記 $\angle AOB = a°$、$\angle BOC = b°$、$\angle COD = c°$、$\angle DOE = d°$ 和 $\angle EOF = e°$。已知 a、b、c、d、e 均爲整數，且存在整數 k，使得 $b = ka$、$c = kb$、$d = kc$ 和 $e = kd$。求 c 的值。（5分）



In the figure, O is the centre of the semi-circle, and A, B, C, D, E, F are points on the circumference with AF being a diameter. Denote $\angle AOB = a°$, $\angle BOC = b°$, $\angle COD = c°$, $\angle DOE = d°$ and $\angle EOF = e°$. Suppose a, b, c, d and e are all integers, and that there exists an integer k, for which $b = ka$, $c = kb$, $d = kc$ and $e = kd$. Find the value of c. (5 marks)

13. 若整數 m 和 n 滿足 $m^3(n^2+1)(n^4-n^2+1) = 1728$，求 m 的所有可能值之和。（6分）

If integers m and n satisfy $m^3(n^2+1)(n^4-n^2+1) = 1728$, find the sum of all possible value(s) of m. (6 marks)

14. 嘉穎購買了 m 個蘋果和 n 個橙（m 和 n 可以爲 0），共付款 96 元。若每個蘋果售價 3 元、每個橙售價 4 元，那麼 (m, n) 有多少組不同的可能值？（6分）

Karina bought m apples and n oranges (m and n can be 0) and paid 96 dollars. If each apple costs 3 dollars while each orange costs 4 dollars, how many possible pairs of (m, n) are there? (6 marks)

15. 若 $\frac{12}{2013} < \frac{m}{n} < \frac{13}{2014}$，其中 m 和 n 爲正整數，求 n 的最小可能值。（6分）

If $\frac{12}{2013} < \frac{m}{n} < \frac{13}{2014}$, where m and n are positive integers, find the smallest possible value of n. (6 marks)

16. 最少需從 1 至 100 這些整數中刪除多少個，才可使餘下各數之積的個位數字爲 2？（6分）

At least how many numbers have to be deleted from the integers from 1 to 100 so that the product of the remaining numbers has unit digit 2? (6 marks)

17. 如果某正整數由左至右和由右至左看皆相同，我們稱這個數爲「回文數」，例如：3883、12321 和 25052 都是「回文數」。若 n 是三位「回文數」，而 n^2 亦是「回文數」，求 n 所有可能值之和。（7 分）

If a positive integer reads the same from left to right as from right to left, it is called a 'palindrome'. For example, 3883, 12321 and 25052 are 'palindromes'. If n is a three-digit 'palindrome' such that n^2 is also a 'palindrome', find the sum of all possible values of n. (7 marks)

18. 在所示的算式中，每個字母代表一個由 1 至 9 的不同數字。求 GHI 所代表的三位數的最大可能值。（7 分）

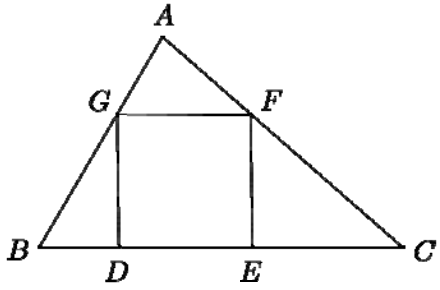
In the addition shown, each letter represents a different digit from 1 to 9. Find the largest possible value of the three-digit number represented by GHI. (7 marks)

$$\begin{array}{cccc} & A & B & C \\ + & D & E & F \\ \hline & G & H & I \end{array}$$

19. 有多少種方法可從 1 至 15 這些整數中選取 4 個不同的數，使得當中任意兩數均不是連續數？（7 分）

How many ways are there to choose 4 distinct numbers from the integers 1 to 15 such that any two chosen numbers are not consecutive? (7 marks)

20. 在 ΔABC 中，D 和 E 爲 BC 上的兩點，F 和 G 則分別爲邊 AC 和 AB 上的點，使得 $DEFG$ 爲正方形。若 ΔAFG 、ΔBDG 和 ΔCEF 的面積分別爲 25、40 和 60，求 BC 的長度。（7 分）



In ΔABC, D and E are two points on BC, while F and G are points on AC and AB respectively such that $DEFG$ is a square. If the areas of ΔAFG, ΔBDG and ΔCEF are 25, 40 and 60 respectively, find the length of BC. (7 marks)

全卷完

END OF PAPER