第十二屆培正數學邀請賽（2013 年）

12th Pui Ching Invitational Mathematics Competition (2013)

初賽（中二組）

Heat Event (Secondary 2)

時限：1 小時 15 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

(a) 本卷共設 20 題，總分爲 100 分。

There are 20 questions in this paper and the total score is 100.

(b) 除特別指明外，本卷內的所有數均爲十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

(c) 作答時，每題的答案均須以 0 至 9999 之間的整數表示。依照答題紙上的指示塡寫答案，毋須呈交計算步驟。

Each answer must be given in the form of an integer between 0 and 9999. Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

(d) 不得使用計算機。

The use of calculators is not allowed.

(e) 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

注意：每題的答案均須以 0 至 9999 之間的整數表示，如有需要應以上述範圍內最接近正確答案的整數回答。如有兩個這樣的整數與正確答案同樣接近，則以「四捨五入」的原則取較大的整數。請細閱答題紙上的指示。

Note: Each answer must be given in the form of an integer between 0 and 9999. Where necessary, the answer should be rounded off to the nearest integer in the above range. Read the instructions on the answer sheet in detail.

1. 在 2013 年中，有多少天的「月」和「日」都是質數？（3 分）

How many days in the year 2013 are there such that the ‘month’ and ‘day’ are both prime numbers? (3 marks)

2. 已知從無窮實數數列 $a_1, a_2, a_3, \ldots$ 中，任意一項或任意連續多項之積均相同。問 a_2 有多少個不同的可能值？（3 分）

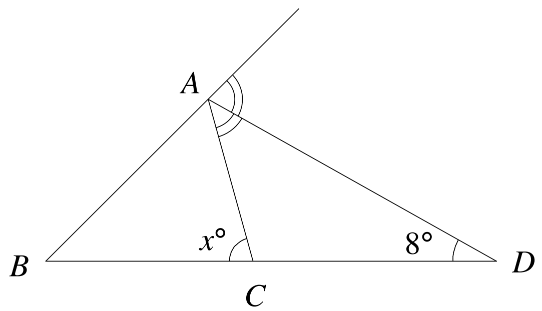
It is known that in the infinite sequence of real numbers $a_1, a_2, a_3, \ldots$, any term and the product of any number of consecutive terms is the same. How many possible values of a_2 are there? (3 marks)

3. 已知 n 爲大於 2013 的整數，且可以表示成三個連續正整數的乘積。求 n 的最小值。（3 分）

Let n be an integer greater than 2013. It is known that n can be expressed as the product of three consecutive positive integers. Find the smallest possible value of n. (3 marks)

4. 在 ΔABC 中，$\angle A$ 爲 $\angle B$ 的兩倍。設 D 爲 $\angle A$ 的外角平分線與 BC 延長線上的交點，且 $\angle ADB = 8^\circ$。若 $\angle ACB = x^\circ$，求 x 的值。（3 分）

In ΔABC, $\angle A$ is twice as large as $\angle B$. Let D be the intersection of the external angle bisector of $\angle A$ and the extension of side BC. Suppose $\angle ADB = 8^\circ$. If $\angle ACB = x^\circ$, find the value of x. (3 marks)



5. 若 x、y 爲正數，且 $2(x^2-3y^2)=xy$，求 $\dfrac{x}{y}$ 的值。（4分）

If x, y are positive and $2(x^2-3y^2)=xy$, find the value of $\dfrac{x}{y}$. (4 marks)

6. 如果某正整數由左至右和由右至左看皆相同，我們稱這個數爲「回文數」。例如 3883、12321 和 25052 都是「回文數」。給定 3000 位數 $\underbrace{11\ldots1}_{1000\text{ 個「1」}}\underbrace{1212\ldots12}_{1000\text{ 組「12」}}$，至少需從該數中刪去多少個數字，才可使餘下的數成爲「回文數」？（4分）

If a positive integer reads the same from left to right as from right to left, it is called a 'palindrome'. For example, 3883, 12321 and 25052 are 'palindromes'. Given the 3000-digit number $\underbrace{11\ldots1}_{1000\text{ digits}}\underbrace{1212\ldots12}_{1000\text{ groups of '12'}}$, at least how many digits must be deleted from the number so that the remaining number is a 'palindrome'? (4 marks)

7. 已知 a 和 n 爲正整數，使得 $a+(a+1)+(a+2)+\cdots+(a+n)=2013$。那麼 a 有多少個不同的可能值？（4分）

Let a and n be positive integers such that $a+(a+1)+(a+2)+\cdots+(a+n)=2013$. How many different possible values of a are there? (4 marks)

8. 現有一張 7×7 的正方形紙，並要把它剪成 49 個單位正方形。每次只能沿某一直線裁剪任意長度，並且不能把方格紙摺起來；唯可以將多於一塊已完全分開的小方格紙疊在一起同時裁剪。那麼整個過程最少需要裁剪多少次？（4分）

A piece of 7×7 square paper is to be cut into 49 unit squares. In each step, we are only allowed to cut along a straight line for any length, and are not allowed to fold the paper. However, we may put more than one piece of paper which has been completely separated over the others and cut them together. At least how many cuts are needed to finish the job? (4 marks)

9. 若 x、y 爲實數，使得 $2x^2+3y^2-4x-12y=-14$，求 xy 的值。（5分）

If x and y are real numbers such that $2x^2+3y^2-4x-12y=-14$, find the value of xy. (5 marks)

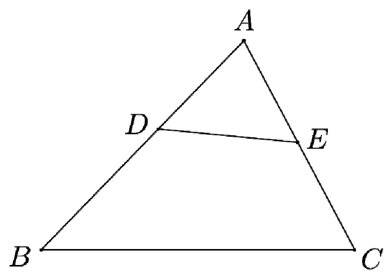
10. 已知對任意正整數 n，n^3+11n 均爲 k 的倍數，其中 k 爲正整數。求 k 所有可能值之和。（5分）

It is given that for any positive integer n, n^3+11n is a multiple of k, where k is a positive integer. Find the sum of all possible values of k. (5 marks)

11. 一隻螞蟻從座標平面的原點 $(0, 0)$ 出發，牠每步可以由點 (x, y) 爬至 $(x+1, y+1)$ 或 $(x+1, y-1)$，問：該螞議爬至點 $(10, 0)$ 有多少種不同方法？（5分）

An ant starts from the origin $(0, 0)$ of the coordinate plane. In each step, it can crawl from (x, y) to either $(x+1, y+1)$ or $(x+1, y-1)$. In how many different ways can the ant crawl to point $(10, 0)$? (5 marks)

12. 在 ΔABC 中，D 和 E 分別是 AB 和 AC 上的點，使得 $AD:DB=2:3$ 及 $AE=EC$。若 ΔADE 的面積爲 120，求 ΔABC 的面積。（5分）



In ΔABC, D and E are points on AB and AC respectively such that $AD:DB=2:3$ and $AE=EC$. If the area of ΔADE is 120, find the area of ΔABC. (5 marks)

13. 小美寫下了一個四位數，它的四位數字都不是 0 而且並非全部相同。小美發現，若把這個數的四位數字任意重新排列，可以得到一些不同的四位數。然後，小美把這些四位數中最小的一個除以最大的一個，得到答案 S。已知 S 的最大可能值以最簡分數表示時爲 $\frac{m}{n}$，求 $n-m$ 的值。（6分）

May wrote down a four-digit number which consists of four non-zero digits that were not all the same. She found that by randomly rearranging the digits, different four-digit numbers can be formed. She then divided the smallest of these numbers by the largest, and obtained the answer S. If the greatest possible value of S is $\frac{m}{n}$ in lowest term, find the value of $n-m$. (6 marks)

14. 在所示的算式中，每個字母代表一個由 0 至 9 的不同數字。求 MATH 所代表的四位數的最大可能值。（6分）

$$\begin{array}{r} 2\ 0\ 1\ 3 \\ J\ A\ N \\ +\qquad 1\ 9 \\ \hline M\ A\ T\ H \end{array}$$

In the addition shown, each letter represents a different digit from 0 to 9. Find the greatest possible value of the four-digit number represented by MATH. (6 marks)

15. 已知 n 是大於 2013 的四位正整數，且當 n 的四位數字左右倒轉時，所得的數亦是一個大於 2013 的四位正整數。問 n 有多少個不同的可能值？（6分）

Given n is a four-digit positive integer larger than 2013. When the four digits of n are reversed, the number obtained is also a four-digit positive integer larger than 2013. How many different possible values of n are there? (6 marks)

16. 三名男孩和三名女孩在周長 2013 米的圓形緩跑徑上緩跑，該緩跑徑上只有一個起跑點。三名男孩分別在起跑點沿順時針方向 100 米、300 米和 500 米處，而三名女孩分別在起跑點沿順時針方向 200 米、400 米和 600 米處。他們同時開始以每秒 1 米的速度緩跑。開始時，男孩都向順時針方向跑，而女孩則向逆時針方向跑。每當任何二人相遇時，他們會立即向相反方向跑，而速度一直保持不變。任何人到達起跑點便會立即離開緩跑徑。若全部人離開緩跑徑需時 x 秒（以開始跑步起計），求 x 的值。（6分）

Three boys and three girls were jogging on a circular jogging track which is 2013 m long. There is only one starting point on the track. The three boys were 100 m, 300 m and 500 m respectively from the starting point clockwise, while the three girls were 200 m, 400 m and 600 m from the starting point clockwise. They started jogging at a speed of 1 m/s at the same time. When they started, boys jogged clockwise while girls jogged anti-clockwise. When any two people met, they would jog in the opposite directions immediately, keeping at the same speed. Anyone reaching the starting point would leave the track at once. Suppose that it took x seconds for all of them to leave the track (counting from the moment they started jogging). Find the value of x. (6 marks)

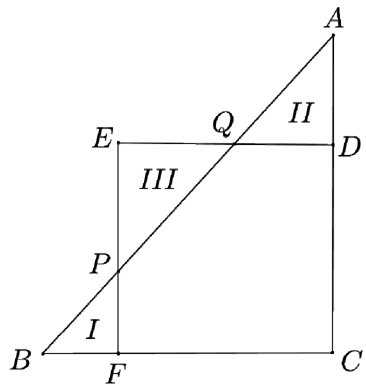
17. 對於正整數 n，設 $S(n)$ 爲 n 的數字之和。若 n 是小於 2013 的正整數，且 $S(n) \geq 5 \cdot S(2n)$，求 n 所有可能值之和。（7分）

For positive integer n, let $S(n)$ denote the sum of digits of n. If n is a positive integer less than 2013 such that $S(n) \geq 5 \cdot S(2n)$, find the sum of all possible values of n. (7 marks)

18. 已知對正整數 n 皆有 $1^2+2^2+\cdots+n^2=\frac{n(n+1)(2n+1)}{6}$。若 $1^2+2^2+\cdots+n^2$ 可被 2013 整除，求 n 的最小可能值。 （7 分）

Given $1^2+2^2+\cdots+n^2=\frac{n(n+1)(2n+1)}{6}$ for positive integer n. If $1^2+2^2+\cdots+n^2$ is divisible by 2013, find the smallest possible value of n. (7 marks)

19. 在 ΔABC 中，$\angle C=90°$，$AC=90$ 及 $BC=80$。已知正方形 $CDEF$ 的頂點 D 和 F 分別在邊 AC 和 BC 上，而點 E 在 ΔABC 外。如圖所示，把 ΔABC 和正方形 $CDEF$ 不重疊的部分分別稱爲區域 I、II 和 III。若區域 I、II 的面積之和等於區域 III 的面積，求正方形 $CDEF$ 的邊長。 （7 分）



In ΔABC, $\angle C=90°$, $AC=90$ and $BC=80$. Suppose the vertices D and F of a square $CDEF$ lie on sides AC and BC respectively, and point E lies outside ΔABC. As shown in the diagram, the non-overlapping areas of ΔABC and the square $CDEF$ are labelled as regions *I*, *II* and *III* respectively. If the sum of areas of regions *I* and *II* is equal to the area of region *III*, find the side length of the square *CDEF*. (7 marks)

20. 某團體訪問了 n 個家庭的母親，得知在她們的子女當中，95 人沒有哥哥，180 人既沒有姊姊，也沒有弟弟（同一名子女可能同時符合兩項條件）。已知任何兩名來自不同家庭的成員均沒有親屬關係，求 n 的最小可能值。 （7 分）

An organisation interviewed the mothers of n families. It was found that among the children of these mothers, 95 of them do not have any elder brothers, 180 of them do not have any elder sisters or younger brothers. (The same child may satisfy both conditions.) It is known that every two members coming from different families are not kin to each other. Find the smallest possible value of n. (7 marks)

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END OF PAPER