

第十二屆培正數學邀請賽（2013 年）

12th Pui Ching Invitational Mathematics Competition (2013)

初賽（中一組）

Heat Event (Secondary 1)

時限：1 小時 15 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

- (a) 本卷共設 20 題，總分爲 100 分。

There are 20 questions in this paper and the total score is 100.

- (b) 除特別指明外，本卷內的所有數均爲十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

- (c) 作答時，每題的答案均須以 0 至 9999 之間的整數表示。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

Each answer must be given in the form of an integer between 0 and 9999. Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

- (d) 不得使用計算機。

The use of calculators is not allowed.

- (e) 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

注意：每題的答案均須以 0 至 9999 之間的整數表示，如有需要應以上述範圍內最接近正確答案的整數回答。如有兩個這樣的整數與正確答案同樣接近，則以「四捨五入」的原則取較大的整數。請細閱答題紙上的指示。

Note: Each answer must be given in the form of an integer between 0 and 9999. Where necessary, the answer should be rounded off to the nearest integer in the above range. Read the instructions on the answer sheet in detail.

1. 已知 p 和 q 為偶數，其中 $p < q$ 且 $p + q = 100$ 。求 p 的最大可能值。 (3 分)

Let p and q be even numbers such that $p < q$ and $p + q = 100$. Find the greatest possible value of p . (3 marks)

2. 在 2013 年中，有多少天的「月」和「日」都是質數？ (3 分)

How many days in the year 2013 are there such that the 'month' and 'day' are both prime numbers? (3 marks)

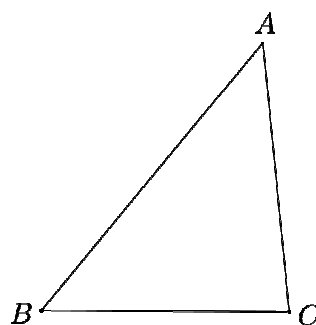
3. 已知 n 為大於 2013 的整數，且可以表示成三個連續正整數的乘積。求 n 的最小值。 (3 分)

Let n be an integer greater than 2013. It is known that n can be expressed as the product of three consecutive positive integers. Find the smallest possible value of n . (3 marks)

4. $\triangle ABC$ 的內角滿足 $\angle A : \angle B : \angle C = 1 : 3 : 4$ 。若 $\angle ABC = x^\circ$ ，求 x 的值。

The angles of $\triangle ABC$ satisfy $\angle A : \angle B : \angle C = 1 : 3 : 4$.

If $\angle ABC = x^\circ$, find the value of x .



5. 若某正整數剛好有 9 個正因數，求此正整數的最小值。 (4 分)

A positive integer has exactly 9 positive factors. Find the smallest possible value of this positive integer. (4 marks)

6. 若某正整數可寫成兩個連續正整數的乘積，那麼我們稱它為「好數」，例如：因為 $8 \times 9 = 72$ ，所以 72 是「好數」。已知 n 是「好數」，它亦可寫成兩個不同的「好數」之和。求 n 的最小可能值。 (4 分)

A positive integer is 'good' if it can be written as the product of two consecutive positive integers. For instance, 72 is 'good' since $8 \times 9 = 72$. It is given that n is a 'good' integer and it can also be expressed as the sum of two distinct 'good' integers. Find the smallest possible value of n . (4 marks)

7. 某正整數除以 7 時餘數為 2，除以 13 時餘數為 4。求此正整數的最小值。 (4 分)

Find the smallest positive integer which leaves a remainder of 2 when divided by 7 and 4 when divided by 13. (4 marks)

8. 若要從一個正方體選取三個頂點，使它們構成一個等腰三角形（包括等邊三角形），共有多少種選法？ (5 分)

In how many ways can three vertices be chosen from a cube so that they form an isosceles triangle (including equilateral triangle)? (5 marks)

9. 設 x 、 y 為有理數，使得 $x^2 - y = 4$ 和 $y^2 - x = 8$ 。求 $x^3 - y^3$ 的值。 (5 分)

Let x and y be rational numbers such that $x^2 - y = 4$ and $y^2 - x = 8$. Find the value of $x^3 - y^3$. (5 marks)

10. 已知 a 和 n 為正整數，使得 $a + (a+1) + (a+2) + \cdots + (a+n) = 100$ 。求 a 所有可能值之和。 (5 分)

Let a and n be positive integers such that $a + (a+1) + (a+2) + \cdots + (a+n) = 100$. Find the sum of all possible values of a . (5 marks)

11. 已知 n 是小於 2013 的四位正整數，且當 n 的四位數字左右倒轉時，所得的數亦是一個小於 2013 的四位正整數。問 n 有多少個不同的可能值？ (5 分)

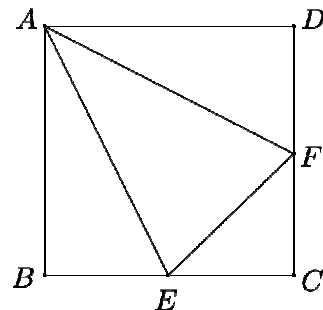
Given n is a four-digit positive integer smaller than 2013. When the four digits of n are reversed, the number obtained is also a four-digit positive integer smaller than 2013. How many different possible values of n are there? (5 marks)

12. 求 $\left(\sqrt{29+2\sqrt{210}}+\sqrt{29-2\sqrt{210}}\right)^2$ 的值。 (5 分)

Evaluate $\left(\sqrt{29+2\sqrt{210}}+\sqrt{29-2\sqrt{210}}\right)^2$. (5 marks)

13. 圖中， $ABCD$ 是正方形， E 、 F 分別是 BC 和 CD 的中點。若 $\triangle AEF$ 的面積為 2013，求正方形 $ABCD$ 的面積。

In the figure, $ABCD$ is a square and E, F are the mid-points of BC and CD respectively. Suppose that the area of $\triangle AEF$ is 2013. Find the area of the square $ABCD$.



(5 分)

(5 marks)

14. 某班有 8 名編號分別為 1 至 8 的學生。現要安排他們圍繞圓桌而坐，使得任意兩名相鄰的學生的編號之和均為質數。問有多少種不同的方法安排座位？（如果每人左方的人在兩種方法中皆相同，則這兩種安排座位的方法視為相同。）

(6 分)

In a class there are 8 students numbered 1 to 8. They are to be seated around a circular table such that the sum of the numbers of any two consecutive students is a prime number. How many different seating arrangements are there? (Two seating arrangements are regarded to be the same if every person finds the same left-hand neighbour in the two arrangements.)

(6 marks)

15. 小美寫下了一個四位數，它的四位數字都不是 0 而且並非全部相同。小美發現，若把這個數的四位數字任意重新排列，可以得到一些不同的四位數。然後，小美把這些四位數中最小的一個除以最大的一個，得到答案 S 。已知 S 的最大可能值以最簡分數表示時為 $\frac{m}{n}$ ，求 $n-m$ 的值。

(6 分)

May wrote down a four-digit number which consists of four non-zero digits that were not all the same. She found that by randomly rearranging the digits, different four-digit numbers can be formed. She then divided the smallest of these numbers by the largest, and obtained the answer S . If the greatest possible value of S is $\frac{m}{n}$ in lowest term, find the value of $n-m$.

(6 marks)

16. 三名男孩和三名女孩在周長 2013 米的圓形緩跑徑上緩跑，該緩跑徑上只有一個起跑點。三名男孩分別在起跑點沿順時針方向 100 米、300 米和 500 米處，而三名女孩分別在起跑點沿順時針方向 200 米、400 米和 600 米處。他們同時開始以每秒 1 米的速度緩跑。開始時，男孩都向順時針方向跑，而女孩則向逆時針方向跑。每當任何二人相遇時，他們會立即向相反方向跑，而速度一直保持不變。任何人到達起跑點便會立即離開緩跑徑。若全部人離開緩跑徑需時 x 秒（以開始跑步起計），求 x 的值。 (6 分)

Three boys and three girls were jogging on a circular jogging track which is 2013 m long. There is only one starting point on the track. The three boys were 100 m, 300 m and 500 m respectively from the starting point clockwise, while the three girls were 200 m, 400 m and 600 m from the starting point clockwise. They started jogging at a speed of 1 m/s at the same time. When they started, boys jogged clockwise while girls jogged anti-clockwise. When any two people met, they would jog in the opposite directions immediately, keeping at the same speed. Anyone reaching the starting point would leave the track at once. Suppose that it took x seconds for all of them to leave the track (counting from the moment they started jogging). Find the value of x . (6 marks)

17. 設 n 為正整數，使得 $1+2+\cdots+n$ 能被 2013 整除。求 n 的最小可能值。 (7 分)

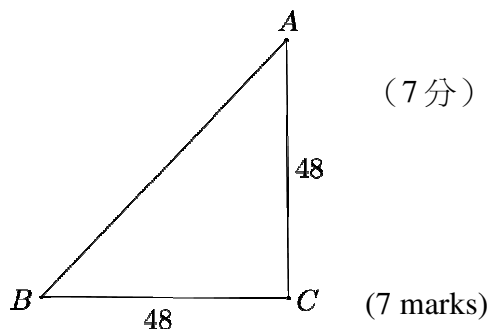
Let n be a positive integer such that $1+2+\cdots+n$ is divisible by 2013. Find the smallest possible value of n . (7 marks)

18. 對於正整數 n ，設 $S(n)$ 為 n 的數字之和。若 n 是小於 2013 的正整數，且 $S(n) \geq 5 \cdot S(2n)$ ，求 n 所有可能值之和。 (7 分)

For positive integer n , let $S(n)$ denote the sum of digits of n . If n is a positive integer less than 2013 such that $S(n) \geq 5 \cdot S(2n)$, find the sum of all possible values of n . (7 marks)

19. 已知 ABC 是直角等腰三角形，其中 $\angle ACB = 90^\circ$ ，且 $AC = BC = 48$ 。某個面積為 S 的正方形的四個頂點均位於 $\triangle ABC$ 的邊上。求 S 所有可能值之和。

Given ABC is a right-angled isosceles triangle with $\angle ACB = 90^\circ$ and $AC = BC = 48$. A square with area S has all its four vertices lying on the sides of $\triangle ABC$. Find the sum of all possible values of S .



20. 某團體訪問了 n 個家庭的母親，得知在她們的子女當中，95 人沒有哥哥，180 人既沒有姊姊，也沒有弟弟（同一名子女可能同時符合兩項條件）。已知任何兩名來自不同家庭的成員均沒有親屬關係，求 n 的最小可能值。 (7 分)

An organisation interviewed the mothers of n families. It was found that among the children of these mothers, 95 of them do not have any elder brothers, 180 of them do not have any elder sisters or younger brothers. (The same child may satisfy both conditions.) It is known that every two members coming from different families are not kin to each other. Find the smallest possible value of n . (7 marks)

全卷完

END OF PAPER