# 第十一屆培正數學邀請賽（2012 年）

# 11th Pui Ching Invitational Mathematics Competition (2012)

## 初賽（高中組）

## Heat Event (Senior Secondary)

**時限：1 小時 15 分**

**Time allowed: 1 hour 15 minutes**

**參賽者須知：**

**Instructions to Contestants:**

(a) 本卷共設 20 題，總分爲 100 分。

There are 20 questions in this paper and the total score is 100.

(b) 除特別指明外，本卷內的所有數均爲十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

(c) 作答時，每題的答案均須以 0 至 9999 之間的整數表示。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

Each answer must be given in the form of an integer between 0 and 9999. Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

(d) 不得使用計算機。

The use of calculators is not allowed.

(e) 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

注意：本屆初賽的規則有所修改，如某題的正確答案並非 0 至 9999 之間的整數，應以上述範圍內最接近正確答案的整數回答。如有兩個這樣的整數與正確答案同樣接近，則以「四捨五入」的原則取較大的整數。請細閱答題紙上的指示。

Note: There have been amendments in the regulations of the current Heat Event. If the correct answer to a question is not an integer between 0 and 9999, one should pick the integer in the above range which is closest to the correct answer. In case of an answer midway between two such integers, round up to the larger integer. Read the instructions on the answer sheet in detail.

1. 思思打算抄寫某等差數列的四個連續項，結果寫下了 11、16、19、23。已知思思抄錯了剛好一個數，則思思寫下的數當中哪一個是錯的？ （3分）

   In copying four consecutive terms of an arithmetic sequence, Grace wrote 11, 16, 19, 23. Given that Grace had copied exactly one number wrongly, which of the numbers she wrote is wrong? (3 marks)

2. 有多少個兩位正整數是 2012 的因數？ （3分）

   How many two-digit positive integers are factors of 2012? (3 marks)

3. 若 $n$ 是正整數，使得 $\log_3 n > 5$，求 $n$ 的最小可能值。 （3分）

   If $n$ is a positive integer such that $\log_3 n > 5$, find the smallest possible value of $n$. (3 marks)

4. 設 $k$ 爲實常數。已知對實數 $x$，$x^2 - 5x + k$ 的最小值爲 2012。求 $k$。 （3分）

   Let $k$ be a real constant. It is given that for real number $x$, the minimum value of $x^2 - 5x + k$ is 2012. Find $k$. (3 marks)

5. 在一個圓內接四邊形中，其中三隻內角的大小分別是 50°、60° 和 $x$°。求 $x$ 所有可能值之和。 （3分）

   In a cyclic quadrilateral, three of the interior angles measure 50°, 60° and $x$°. Find the sum of all possible values of $x$. (3 marks)

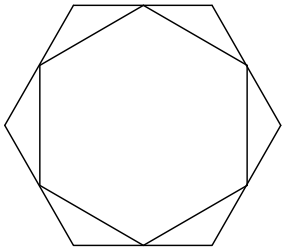
6. 若 $f(x) = \cos 2x$，求 $f^{(12)}(0)$ 的值。（這裡 $f^{(n)}$ 表示 $f$ 的 $n$ 階導數。） （3分）

   If $f(x) = \cos 2x$, find the value of $f^{(12)}(0)$. (Here $f^{(n)}$ represents the $n$-th derivative of $f$.) (3 marks)

7. 希桐把兩個兩位數相乘時，不小心把其中一個數的兩位數字倒轉了，結果得出錯誤答案 1888，比正確答案大。正確答案是多少？ （4 分）

When multiplying two two-digit numbers, Tiffany carelessly swapped the two digits of one of the numbers, resulting in the wrong answer 1888 which is greater than the correct answer. What is the correct answer? (4 marks)

8. 把一個正六邊形各邊的中點連起，可以得到一個較小的正六邊形。這個小六邊形的面積是原六邊形的百分之幾？ （4 分）



By joining the mid-points of a regular hexagon, one obtains a smaller regular hexagon. What percent of the area of the original hexagon is the area of the smaller hexagon? (4 marks)

9. 求最接近 $(2.012)^6$ 的整數。 （4 分）

Find the integer closest to $(2.012)^6$. (4 marks)

10. 設 $n$ 爲正整數。若 $n$、$n+1$ 和 $n+2$ 各有剛好四個正因數，求 $n$ 的最小可能值。 （5 分）

Let $n$ be a positive integer. If each of $n$, $n+1$ and $n+2$ has exactly four positive factors, find the smallest possible value of $n$. (5 marks)

11. 有多少個四位正整數的數字之積是 0？ （5 分）

How many four-digit positive integers have their product of digits being 0? (5 marks)

12. 如果某正整數由左至右和由右至左看皆相同，我們稱這個數爲「回文數」。例如 3883、12321 和 25052 都是「回文數」。若 $n$ 和 $n+3210$ 都是「回文數」，求 $n$ 的最小可能值。 （6 分）

If a positive integer reads the same from left to right as from right to left, it is called a ‘palindrome’. For example, 3883, 12321 and 25052 are ‘palindromes’. If $n$ and $n+3210$ are both ‘palindromes’, find the smallest possible value of $n$. (6 marks)

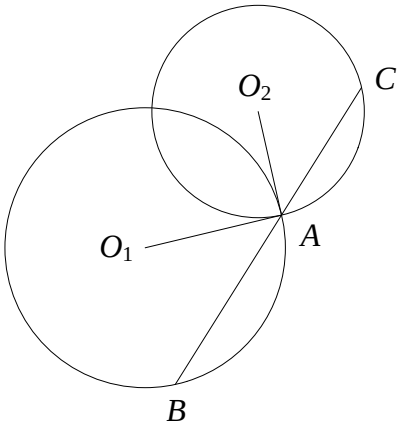
13. 求 $\int_{-13}^{5} \sqrt{65-8x-x^2}\ dx$ 的值。 （6分）

Evaluate $\int_{-13}^{5} \sqrt{65-8x-x^2}\ dx$. (6 marks)

14. 若 $2013\sin^2 x° - 2\sin x°\cos x° + 2011\cos^2 x° = 2012$ ，其中 $0 < x < 90$ ，求 $x$。 （6分）

If $2013\sin^2 x° - 2\sin x°\cos x° + 2011\cos^2 x° = 2012$ where $0 < x < 90$, find $x$. (6 marks)

15. 如圖所示，$O_1$ 和 $O_2$ 是兩圓的圓心，而兩圓的半徑分別是 20 和 10。$A$ 是兩圓的其中一個交點，而 $\angle O_1AO_2 = 120°$。一條穿過 $A$ 的線段與兩圓分別交於 $B$ 和 $C$。求 $BC$ 的長度的最大可能值。 （7分）




In the figure, $O_1$ and $O_2$ are the centres of the two circles whose radii are 20 and 10 respectively. $A$ is one of the points of intersection of the two circles and $\angle O_1AO_2 = 120°$. A line segment through $A$ meets the two circles at $B$ and $C$ respectively. Find the greatest possible length of $BC$. (7 marks)

16. 在所示的算式中，每個字母代表一個由 0 至 9 的不同數字。求 ABC 所代表的三位數。 （7分）

In the addition shown, each letter represents a different digit from 0 to 9. Find the three-digit number represented by ABC. (7 marks)

$$\begin{array}{rcccccc} & & & A & B & C \\ \times & & & C & B & A \\ \hline & G & J & G & C & \\ & & G & H & I & F \\ & & & D & E & B & F \\ \hline & G & B & G & G & B & F \end{array}$$

17. 現於一個圓的圓周上選 20 點，並且每兩點均以一條直線連起。那麼，最多可在圓的內部形成多少個交點？ （7分）

20 points are marked on the circumference of a circle. If every two points are joined by a straight line, what is the maximum number of points of intersections formed in the interior of the circle? (7 marks)

18. 設 $a$、$b$、$c$ 爲正整數，使得 $a<b<c$。若 $a+b$、$b+c$ 和 $c+a$ 都是立方數，求 $c$ 的最小可能值。（7分）

Let $a$, $b$, $c$ be positive integers with $a<b<c$. If $a+b$, $b+c$ and $c+a$ are all cubic numbers, find the smallest possible value of $c$. (7 marks)

19. 嘉永有四件不同的外套，他每天都會隨意穿上其中一件。已知在某一星期中最少一件外套被穿上最少三次的概率以最簡分數表示爲 $\frac{a}{b}$，求 $a+b$。（7分）

Calvin has 4 different coats and he randomly chooses one to put on every day. If the probability that at least one coat has been put on at least three times in a certain week is $\frac{a}{b}$ when expressed as a fraction in lowest term, find $a+b$. (7 marks)

20. 已知對任意非零常數 $k$ 皆有 $\lim\limits_{x\to 0}\frac{1-(\cos kx)^{\frac{1}{k}}}{x^2}=\frac{k}{2}$。求以下極限的值：（7分）

$$\lim_{x\to 0}\frac{1-(\cos x)(\cos 2x)^{\frac{1}{2}}(\cos 3x)^{\frac{1}{3}}\cdots(\cos 100x)^{\frac{1}{100}}}{x^2}$$

It is known that $\lim\limits_{x\to 0}\frac{1-(\cos kx)^{\frac{1}{k}}}{x^2}=\frac{k}{2}$ for any non-zero constant $k$. Evaluate the following limit: (7 marks)

$$\lim_{x\to 0}\frac{1-(\cos x)(\cos 2x)^{\frac{1}{2}}(\cos 3x)^{\frac{1}{3}}\cdots(\cos 100x)^{\frac{1}{100}}}{x^2}$$

再次提醒各位參賽者，作答時每題的答案均須以 0 至 9999 之間的整數表示，如有需要應以上述範圍內最接近正確答案的整數回答。請細閱答題紙上的指示。

Would contestants please be reminded again that each answer must be given in the form of an integer between 0 and 9999. Where necessary, the answer should be rounded off to the nearest integer in the above range. Read the instructions on the answer sheet in detail.

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**END OF PAPER**