第十一屆培正數學邀請賽（2012 年）

11th Pui Ching Invitational Mathematics Competition (2012)

初賽（中四組）

Heat Event (Secondary 4)

時限：**1** 小時 **15** 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

(a) 本卷共設 20 題，總分為 100 分。

There are 20 questions in this paper and the total score is 100.

(b) 除特別指明外，本卷內的所有數均為十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

(c) 作答時，每題的答案均須以 0 至 9999 之間的整數表示。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

Each answer must be given in the form of an integer between 0 and 9999. Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

(d) 不得使用計算機。

The use of calculators is not allowed.

(e) 本卷的附圖不一定依比例繪成。

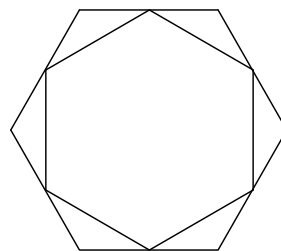The diagrams in this paper are not necessarily drawn to scale.

注意：本屆初賽的規則有所修改，如某題的正確答案並非 0 至 9999 之間的整數，應以上述範圍內最接近正確答案的整數回答。如有兩個這樣的整數與正確答案同樣接近，則以「四捨五入」的原則取較大的整數。請細閱答題紙上的指示。

Note: There have been amendments in the regulations of the current Heat Event. If the correct answer to a question is not an integer between 0 and 9999, one should pick the integer in the above range which is closest to the correct answer. In case of an answer midway between two such integers, round up to the larger integer. Read the instructions on the answer sheet in detail.

1. 設 k 為正整數。若 20120204 可被 2^k 整除，求 k 的最大可能值。 （3分）

   Let k be a positive integer. If 20120204 is divisible by 2^k, find the greatest possible value of k. (3 marks)

2. 在一個 100 項的數列中，任意連續三項之和皆是 10。若全部 100 項之和為 2012，求首項。 （3分）

   In a sequence of 100 terms, the sum of any three consecutive terms is equal to 10. If the sum of all 100 terms is 2012, find the first term. (3 marks)

3. 方程 $x^{13} - 7x^7 + 10x = 0$ 有多少個不同的實根？ （4分）

   How many different real roots does the equation $x^{13} - 7x^7 + 10x = 0$ have? (4 marks)

4. 求 $\log_5 2012$ 的值。 （4分）

   Find the value of $\log_5 2012$. (4 marks)

5. 希桐把兩個兩位數相乘時，不小心把其中一個數的兩位數字倒轉了，結果得出錯誤答案 1888，比正確答案大。正確答案是多少？ （4分）

   When multiplying two two-digit numbers, Tiffany carelessly swapped the two digits of one of the numbers, resulting in the wrong answer 1888 which is greater than the correct answer. What is the correct answer? (4 marks)

6. 某圓形的面積為 2012。求它的半徑。 （4分）

   A circle has area 2012. Find its radius. (4 marks)

7. 把一個正六邊形各邊的中點連起，可以得到一個較小的正六邊形。這個小六邊形的面積是原六邊形的百分之幾？ （4分）

   By joining the mid-points of a regular hexagon, one obtains a smaller regular hexagon. What percent of the area of the original hexagon is the area of the smaller hexagon? (4 marks)

8. 設 a、b、c、d、e、f、g 為從小至大排列的連續正整數。若這七個數之和是立方數，求 g 的最小可能值。 （4分）

   Let a, b, c, d, e, f, g be consecutive positive integers arranged in ascending order. If the sum of the seven numbers is a cubic number, find the smallest possible value of g. (4 marks)

9. 設 a、b、c、d、e 為從小至大排列的連續正整數。若這五個數都是合成數，求 a 的最小可能值。 （4分）

   Let a, b, c, d, e be consecutive positive integers arranged in ascending order. If all the five numbers are composite, find the smallest possible value of a. (4 marks)

10. 一次派對有 4 名男孩和 n 名女孩參加。他們當中有些人曾經互相握手，但每人都只會和異性握手，而且每兩人之間最多握手一次。已知 4 名男孩的握手次數分別為 25、30、35 和 40，而每名女孩的握手次數最多為 3，求 n 的最小可能值。 （5分）

    In a party, there are 4 boys and n girls. Some of them have shaken hands with each other; nobody shakes hand with people of the same gender and every pair of participants shake hands at most once. The 4 boys have shaken hands 25, 30, 35 and 40 times respectively, while each girl has shaken hands at most 3 times. Find the smallest possible value of n. (5 marks)

11. 圖中，$ABCD$ 是圓內接四邊形，其面積為 S。若 $AB = 5$、$BC = CD = 4$ 而 $\angle B = 60°$，求 S^2。 （5分）

    In the figure, $ABCD$ is a cyclic quadrilateral with area S. If $AB = 5$, $BC = CD = 4$ and $\angle B = 60°$, find S^2. (5 marks)

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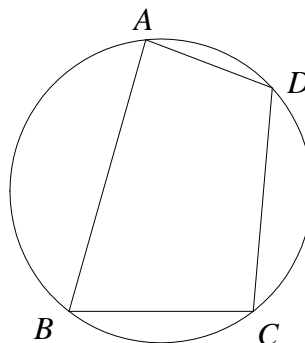
12. 一張圓桌的外圍有五個不同顏色的座位。現要安排五名編號為 1 至 5 的學生繞圓桌就坐，使得相鄰的學生的編號最少相差 2。問有多少種不同的方法安排座位？ （5分）

At a round table there are five seats with different colours. In how many different ways can we arrange five students numbered 1 to 5 to be seated around the table so that adjacent students have their numbers differing by at least 2? (5 marks)

13. 若 $2013\sin^2 x° - 2\sin x°\cos x° + 2011\cos^2 x° = 2012$，其中 $0 < x < 90$，求 x。 （6分）

If $2013\sin^2 x° - 2\sin x°\cos x° + 2011\cos^2 x° = 2012$ where $0 < x < 90$, find x. (6 marks)

14. 如果某正整數由左至右和由右至左看皆相同，我們稱這個數為「回文數」。例如 3883、12321 和 25052 都是「回文數」。若 n 和 $n+3210$ 都是「回文數」，求 n 的最小可能值。 （6分）

If a positive integer reads the same from left to right as from right to left, it is called a 'palindrome'. For example, 3883, 12321 and 25052 are 'palindromes'. If n and $n+3210$ are both 'palindromes', find the smallest possible value of n. (6 marks)

15. 七個兩位數的平均數、中位數和唯一的眾數是互不相同的正整數。求該平均數的最小可能值。 （6分）

The mean, median and unique mode of seven two-digit numbers are pairwise different positive integers. Find the smallest possible value of the mean. (6 marks)

16. 一個特別的二十四小時制時鐘以四位數字表示時間，例如凌晨十二時半以「0030」表示，下午八時十二分則以「2012」表示。某天正午，時鐘被調準，然後時鐘需經過 61 秒才會跳至下一分鐘（即在 61 秒後，它的顯示會由「1200」變成「1201」），之後再需經過 62 秒才會跳至再下一分鐘，如此類推。在之後一天的正午，時鐘上顯示的時間是甚麼？ （6分）

A special clock uses four digits to show the time in 24-hour mode. For example half past twelve at midnight is shown by '0030' while twelve past eight at night is shown by '2012'. At noon of a certain day the clock is set to the exact time. It then takes 61 seconds to lapse to the next minute (i.e. after 61 seconds the display will switch from '1200' to '1201'), 62 seconds to switch to yet the next minute, and so on. What time will be displayed on the clock at noon on the subsequent day? (6 marks)

17. 設 $p(x)$ 爲多項式，其所有系數均爲非負整數。若 $p(1) = 4$ 而 $p(7) = 2500$，求 $p(4)$。　　　　　　　　　　　　　　　　　　　　　　　　　　（6分）

Let $p(x)$ be a polynomial in which all coefficients are non-negative integers. If $p(1) = 4$ and $p(7) = 2500$, find $p(4)$.　　　　　　　　（6 marks）

18. 如圖所示，O_1 和 O_2 是兩圓的圓心，而兩圓的半徑分別是 20 和 10。A 是兩圓的其中一個交點，而 $\angle O_1AO_2 = 120°$。一條穿過 A 的線段與兩圓分別交於 B 和 C。求 BC 的長度的最大可能值。　　　　　　　　　　（7分）

In the figure, O_1 and O_2 are the centres of the two circles whose radii are 20 and 10 respectively. A is one of the points of intersection of the two circles and $\angle O_1AO_2 = 120°$. A line segment through A meets the two circles at B and C respectively. Find the greatest possible length of BC.　　　　（7 marks）

19. 在所示的算式中，每個字母代表一個由 0 至 9 的不同數字。求 ABC 所代表的三位數。　　　　　　（7分）

In the addition shown, each letter represents a different digit from 0 to 9. Find the three-digit number represented by ABC.

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        A B C
  ×     C B A
  ─────────────
      G J G C
      G H I F
      D E B F
  ─────────────
    G B G G B F
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（7 marks）

20. 嘉永有四件不同的外套，他每天都會隨意穿上其中一件。已知在某一星期中最少一件外套被穿上最少三次的概率以最簡分數表示爲 $\dfrac{a}{b}$，求 $a+b$。　　　　（7分）

Calvin has 4 different coats and he randomly chooses one to put on every day. If the probability that at least one coat has been put on at least three times in a certain week is $\dfrac{a}{b}$ when expressed as a fraction in lowest term, find $a+b$.　　　　（7 marks）

再次提醒各位參賽者，作答時每題的答案均須以 0 至 9999 之間的整數表示，如有需要應以上述範圍內最接近正確答案的整數回答。請細閱答題紙上的指示。

Would contestants please be reminded again that each answer must be given in the form of an integer between 0 and 9999. Where necessary, the answer should be rounded off to the nearest integer in the above range. Read the instructions on the answer sheet in detail.

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END OF PAPER