第十一屆培正數學邀請賽（2012 年）

11th Pui Ching Invitational Mathematics Competition (2012)

初賽（中三組）

Heat Event (Secondary 3)

時限：1 小時 15 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

(a) 本卷共設 20 題，總分爲 100 分。

There are 20 questions in this paper and the total score is 100.

(b) 除特別指明外，本卷內的所有數均爲十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

(c) 作答時，每題的答案均須以 0 至 9999 之間的整數表示。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

Each answer must be given in the form of an integer between 0 and 9999. Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

(d) 不得使用計算機。

The use of calculators is not allowed.

(e) 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

注意：本屆初賽的規則有所修改，如某題的正確答案並非 0 至 9999 之間的整數，應以上述範圍內最接近正確答案的整數回答。如有兩個這樣的整數與正確答案同樣接近，則以「四捨五入」的原則取較大的整數。請細閱答題紙上的指示。

Note: There have been amendments in the regulations of the current Heat Event. If the correct answer to a question is not an integer between 0 and 9999, one should pick the integer in the above range which is closest to the correct answer. In case of an answer midway between two such integers, round up to the larger integer. Read the instructions on the answer sheet in detail.

1. 如果某正整數由左至右和由右至左看皆相同，我們稱這個數爲「回文數」。例如 3883、12321 和 25052 都是「回文數」。求最接近 2012 的「回文數」。 （3 分）

   If a positive integer reads the same from left to right as from right to left, it is called a 'palindrome'. For example, 3883, 12321 and 25052 are 'palindromes'. Find the 'palindrome' closest to 2012. (3 marks)

2. 求 20120204^2 的最後四位數字。 （3 分）

   Find the last four digits of 20120204^2. (3 marks)

3. 如果某個星期六的日期中的「日」是質數，則這天稱爲「好日子」。在一個月裏，最多有多少天「好日子」？ （3 分）

   If a Saturday falls on a date in which the 'day' is prime, it is said to be a 'good' day. What is the maximum number of 'good' days in a month? (3 marks)

4. 某正方形的面積是 50。求它的對角線的長度。 （3 分）

   A square has area 50. Find the length of its diagonal. (3 marks)

5. 求可被 12、18 和 24 整除的最小四位正整數。 （3 分）

   Find the smallest four-digit positive integer which is divisible by 12, 18 and 24. (3 marks)

6. 兩個正整數相除時，商是正整數，餘數是 2012。求被除數的最小可能值。 （4 分）

   When a positive integer is divided by another positive integer, the quotient is a positive integer and the remainder is 2012. Find the smallest possible value of the dividend. (4 marks)

7. 現有兩個大小不同的圓錐體。若大圓錐體的底半徑是小圓錐體的 3 倍，高是小圓錐體的 4 倍，則其體積是小圓錐體的多少倍？ （4 分）

There are two cones with different sizes. If the base radius of the larger cone is 3 times that of the smaller cone while the height of the larger cone is 4 times that of the smaller cone, how many times the volume of the smaller cone is the volume of the larger cone? (4 marks)

8. 在所示的算式中，每個字母代表一個由 0 至 9 的不同數字。求 CDE 所代表的三位數的最大可能值。 （5 分）

In the expression shown, each letter represents a different digit from 0 to 9. Find the greatest possible value of the three-digit number represented by CDE. (5 marks)

$$\begin{array}{rcccc} & A & B & C \\ & & B & C & D \\ + & & C & D & E \\ \hline & 2 & 0 & 1 & 2 \end{array}$$

9. 一家百貨公司正在進行減價推廣，顧客購買兩件或以上貨品，即可選擇享有八折價惠或免費購得最便宜的一件貨品。小冰計劃購買兩件價錢不同的貨品，她發現無論她選擇那個方案，需付的金額都相同。假設所有貨品的售價（以元爲單位）均爲整數且不超過 2012，問小冰計劃購買的貨品中，較貴的一件的售價有多少個不同的可能值？ （5 分）

A department store is having a sales promotion. Customers purchasing two or more items may choose to have 20% off or get the lowest-priced item free. Pinky plans to buy two items of different prices. She finds that the total amount she has to pay is the same regardless of her choice. If the prices (in dollars) of all goods are integers and not exceeding 2012, how many different possible values are there for the price of the more expensive item Pinky plans to buy? (5 marks)

10. 某國家只有面值 5 元和 8 元的兩種硬幣。若要付款剛好 2012 元，且不設找贖，最少要用幾個 5 元硬幣？ （5 分）

In a country there are only two types of coins, with denominations 5 dollars and 8 dollars respectively. What is the minimum number of 5-dollar coins that must be used in order to pay exactly 2012 dollars without change? (5 marks)

11. 某班學生進行測驗。測驗後，老師決定給每名女生額外加 3 分，結果全班的平均分因而提高了 1.2 分。問班中男生佔百分之幾？ （5 分）

A class of students had a test. After the test, the teacher decided to award an extra 3 marks to each girl. As a result, the average score of the class was increased by 1.2 marks. How many percent of the students of the class are boys? (5 marks)

12. 已知 $\sqrt{2012} \approx 44.8553$。求最接近 $\sqrt[4]{20120204}$ 的整數。 （6 分）

Given $\sqrt{2012} \approx 44.8553$. Find the integer closest to $\sqrt[4]{20120204}$. (6 marks)

13. 一個 2012 邊形中，最多有幾隻角是銳角？ （6 分）

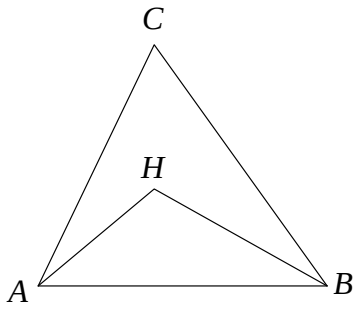
What is the maximum number of acute angles in a 2012-sided polygon? (6 marks)

14. 有多少個四位正整數的十位比個位大，且千位比百位大？ （6 分）

How many four-digit positive integers have the property that the tens digit is greater than the unit digit, and the thousands digit is greater than the hundreds digit? (6 marks)

15. H 是 ΔABC 的垂心。若 $\angle CAB = 56°$ 而 $\angle HAB + \angle HBC = x°$，求 x。 （6 分）

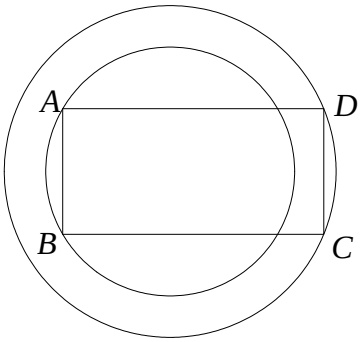
H is the orthocentre of ΔABC. If $\angle CAB = 56°$ and $\angle HAB + \angle HBC = x°$, find x. (6 marks)



16. 設 m 和 n 爲正整數。若 $m+2$ 是 n 的倍數而 $n+2$ 是 m 的倍數，求 n 所有可能值之和。 （6 分）

Let m and n be positive integers. If $m+2$ is a multiple of n while $n+2$ is a multiple of m, find the sum of all possible values of n. (6 marks)

17. 圖中顯示兩個同心圓，它們的半徑分別是 15 和 20。*A*、*B* 是內圓上的兩點，*C*、*D* 是外圓上的兩點。若 *ABCD* 是長方形，求 *ABCD* 的面積的最大可能值。（6 分）



In the figure, the two circles are concentric. Their radii are 15 and 20 respectively. *A*, *B* are points on the inner circle while *C*, *D* are points on the outer circle. If *ABCD* is a rectangle, find the greatest possible area of *ABCD*. (6 marks)

18. 展開並化簡 $(1+x^2+x^4+\cdots+x^{100})^2-(x+x^3+x^5+\cdots+x^{99})^2$ 後，有多少項的系數非零？（7 分）

When $(1+x^2+x^4+\cdots+x^{100})^2-(x+x^3+x^5+\cdots+x^{99})^2$ is expanded and simplified, how many terms have non-zero coefficients? (7 marks)

19. 可怡和樂希各自從首 100 個正整數當中隨意選出一個。若兩人所選的數相差不超過 5 的概率是 $\frac{k}{10000}$，求 k。（7 分）

Chloe and Queenie each randomly picks one out of the first 100 positive integers. If the probability that the two picked numbers differ by no more than 5 is $\frac{k}{10000}$, find k. (7 marks)

20. 小明和大明二人玩遊戲。開始時，兩人的分數均是 0。之後每個回合，如果小明勝出可得 4 分，大明勝出可得 7 分，而且每個回合均由剛好一人勝出。先累積得到 2012 分或以上者勝出遊戲。如果某一回合結束後，原先落後的一方變成領先，我們說遊戲出現了「逆轉」（落後的一方先追至平手再領先不算「逆轉」）。在遊戲過程中，最多會出現多少次「逆轉」？（7 分）

Aaron and Baron play a game. They both start with 0 point. In each round, Aaron gets 4 points if he wins, while Baron gets 7 points if he wins. There is exactly one winner for each round. The first one who accumulates 2012 points or more wins the game. If the side who is originally lagging behind becomes ahead of the other after a round, we say that a 'reversal' has occurred. (If the side behind becomes level with the other side first and then gets ahead, this is not regarded as a 'reversal'.) What is the maximum number of 'reversals' in the game? (7 marks)

再次提醒各位參賽者，作答時每題的答案均須以 0 至 9999 之間的整數表示，如有需要應以上述範圍內最接近正確答案的整數回答。請細閱答題紙上的指示。

Would contestants please be reminded again that each answer must be given in the form of an integer between 0 and 9999. Where necessary, the answer should be rounded off to the nearest integer in the above range. Read the instructions on the answer sheet in detail.

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