第十一屆培正數學邀請賽（2012 年）

11th Pui Ching Invitational Mathematics Competition (2012)

初賽（中二組）

Heat Event (Secondary 2)

時限：1 小時 15 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

(a) 本卷共設 20 題，總分爲 100 分。

There are 20 questions in this paper and the total score is 100.

(b) 除特別指明外，本卷內的所有數均爲十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

(c) 作答時，每題的答案均須以 0 至 9999 之間的整數表示。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

Each answer must be given in the form of an integer between 0 and 9999. Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

(d) 不得使用計算機。

The use of calculators is not allowed.

(e) 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

注意：本屆初賽的規則有所修改，如某題的正確答案並非 0 至 9999 之間的整數，應以上述範圍內最接近正確答案的整數回答。如有兩個這樣的整數與正確答案同樣接近，則以「四捨五入」的原則取較大的整數。請細閱答題紙上的指示。

Note: There have been amendments in the regulations of the current Heat Event. If the correct answer to a question is not an integer between 0 and 9999, one should pick the integer in the above range which is closest to the correct answer. In case of an answer midway between two such integers, round up to the larger integer. Read the instructions on the answer sheet in detail.

1. 若兩個正整數之和是 2012，它們之積的最小可能值是甚麼？ （3 分）

   If the sum of two positive integers is 2012, what is the smallest possible value of their product? (3 marks)

2. 有多少個兩位正整數的十位和個位不同？ （3 分）

   How many two-digit positive integers have their tens digit and unit digit different? (3 marks)

3. 若把 $(a-b)(p-q-r-s)(x-y-z)$ 展開，共會有多少項？ （3 分）

   How many terms will there be when $(a-b)(p-q-r-s)(x-y-z)$ is expanded? (3 marks)

4. 如果某正整數由左至右和由右至左看皆相同，我們稱這個數爲「回文數」。例如 3883、12321 和 25052 都是「回文數」。若 n 是大於 2012 的「回文數」，求 n 的最小可能值。 （3 分）

   If a positive integer reads the same from left to right as from right to left, it is called a 'palindrome'. For example, 3883, 12321 and 25052 are 'palindromes'. If n is a 'palindrome' greater than 2012, find the smallest possible value of n. (3 marks)

5. 某正方形的面積是 50。求它的對角線的長度。 （3 分）

   A square has area 50. Find the length of its diagonal. (3 marks)

6. 現有一些載重量上限爲 7 公斤的箱子。若要盛載 5 件各重 3 公斤和 8 件各重 4 公斤的物品，最少要使用幾個這樣的箱子？（每件物品不能分拆。） （4 分）

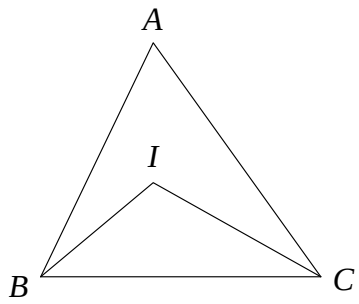
   There are some boxes which can support a maximum load of 7 kg. To carry 5 objects each with weight 3 kg and 8 objects each with weight 4 kg, what is the minimum number of such boxes needed? (Each object cannot be subdivided.) (4 marks)

7. 一家百貨公司正在進行減價推廣，顧客購買兩件或以上貨品，即可選擇享有八折價惠或免費購得最便宜的一件貨品。小冰打算購買四件價值分別爲 12、345、456 和 567 元的貨品，那麼小冰最少要付多少元？ （4 分）

A department store is having a sales promotion. Customers purchasing two or more items may choose to have 20% off or get the lowest-priced item free. Pinky plans to buy four items which cost 12, 345, 456 and 567 dollars respectively. What is the lowest price (in dollars) that Pinky has to pay? (4 marks)

8. 在 ΔABC 中，$\angle B$ 和 $\angle C$ 的內角平分線交於 I。若 $\angle A = 56°$ 而 $\angle IBC + \angle ICA = x°$，求 x。 （4 分）

In ΔABC, the internal bisectors of $\angle B$ and $\angle C$ meet at I. If $\angle A = 56°$ and $\angle IBC + \angle ICA = x°$, find x. (4 marks)



9. 現有四張分別寫上「2」、「0」、「1」、「2」的卡片。若把它們重新排列，共可組成多少個不同的四位正整數？ （5 分）

There are four cards, on which ‘2’, ‘0’, ‘1’, ‘2’ are printed respectively. How many different four-digit positive integers can be formed by rearranging these cards? (5 marks)

10. 在一個凹多邊形中，最大的兩隻內角爲 $x°$ 和 $150°$。若其餘內角之和爲 $2012°$，求 x。 （5 分）

In a concave polygon, the sizes of the two largest interior angles are $x°$ and $150°$. If the sum of the other interior angles is $2012°$, find x. (5 marks)

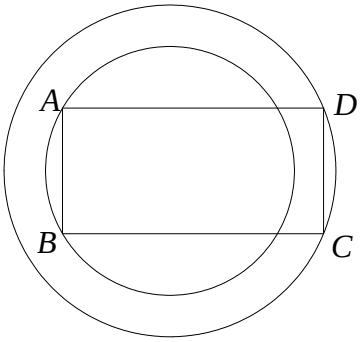
11. 當 2012 位數 201220122012…2012 除以 7 時，餘數是多少？ （5 分）

What is the remainder when the 2012-digit number 201220122012…2012 is divided by 7? (5 marks)

12. 已知 $\sqrt{2012} \approx 44.8553$。若某個八位平方數的首四位是 2012，它的最後四位（從左至右）是甚麼？ （6 分）

Given $\sqrt{2012} \approx 44.8553$. If the first four digits of an eight-digit square number are 2012, what are the last four digits (from left to right)? (6 marks)

13. 圖中顯示兩個同心圓，它們的半徑分別是 15 和 20。*A*、*B* 是內圓上的兩點，*C*、*D* 是外圓上的兩點。若 *ABCD* 是長方形，求 *ABCD* 的面積的最大可能值。 （6 分）



In the figure, the two circles are concentric. Their radii are 15 and 20 respectively. *A*, *B* are points on the inner circle while *C*, *D* are points on the outer circle. If *ABCD* is a rectangle, find the greatest possible area of *ABCD*. (6 marks)

14. 已知 *n* 是一個三位正整數，其中任意兩個數字加起來都是偶數。問 *n* 有多少個不同的可能值？ （6 分）

Given *n* is a three-digit positive integer such that the sum of any two of its digits is even. How many different possible values of *n* are there? (6 marks)

15. 設 *m* 和 *n* 爲正整數。若 $m+2$ 是 *n* 的倍數而 $n+2$ 是 *m* 的倍數，求 *n* 所有可能值之和。 （6 分）

Let *m* and *n* be positive integers. If $m+2$ is a multiple of *n* while $n+2$ is a multiple of *m*, find the sum of all possible values of *n*. (6 marks)

16. 一個 20 邊形中，最多有幾隻角是銳角？ （6 分）

What is the maximum number of acute angles in a 20-sided polygon? (6 marks)

17. 展開並化簡 $(1+x^2+x^4+\cdots+x^{100})^2-(x+x^3+x^5+\cdots+x^{99})^2$ 後，有多少項的系數非零？ （7 分）

When $(1+x^2+x^4+\cdots+x^{100})^2-(x+x^3+x^5+\cdots+x^{99})^2$ is expanded and simplified, how many terms have non-zero coefficients? (7 marks)

18. 在所示的乘式中，每個字母代表一個由 0 至 5 的不同數字。求 ABC 所代表的三位數的所有可能值之和。 （7 分）

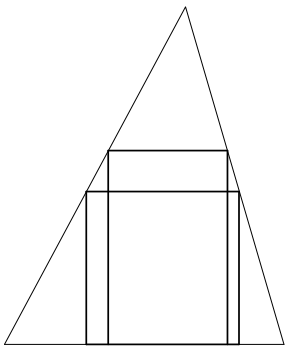
$$\begin{array}{rr} & A\ B\ C \\ \times & C\ B\ A \\ \hline D\ B\ E & E\ B\ F \end{array}$$

In the multiplication shown, each letter represents a different digit from 0 to 5. Find the sum of all possible values of the three-digit number represented by ABC. (7 marks)

19. 現要安排三名女孩和四名男孩圍著一張圓桌就坐。若任意兩名女孩都不可相鄰而坐，則安排座位的方法有多少種？（如果每人左方的人在兩種方法中皆相同，則這兩種安排座位的方法視爲相同。） （7分）

Three girls and four boys are seated at a round table. How many different seating arrangements are there if no two girls may sit next to each other? (Two seating arrangements are regarded to be the same if every person finds the same left-hand neighbour in the two arrangements.) (7 marks)

20. 圖中，一個邊長爲 28 的正方形和一個闊 20、高 32 的長方形內接於同一個三角形。求該三角形的面積。 （7分）



In the figure, a square of side length 28 and a rectangle with width 20 and height 32 are inscribed in the same triangle. Find the area of the triangle. (7 marks)

再次提醒各位參賽者，作答時每題的答案均須以 0 至 9999 之間的整數表示，如有需要應以上述範圍內最接近正確答案的整數回答。請細閱答題紙上的指示。

Would contestants please be reminded again that each answer must be given in the form of an integer between 0 and 9999. Where necessary, the answer should be rounded off to the nearest integer in the above range. Read the instructions on the answer sheet in detail.

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