第十一屆培正數學邀請賽（2012 年）

11th Pui Ching Invitational Mathematics Competition (2012)

初賽（中一組）

Heat Event (Secondary 1)

時限：1 小時 15 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

(a) 本卷共設 20 題，總分爲 100 分。

There are 20 questions in this paper and the total score is 100.

(b) 除特別指明外，本卷內的所有數均爲十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

(c) 作答時，每題的答案均須以 0 至 9999 之間的整數表示。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

Each answer must be given in the form of an integer between 0 and 9999. Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

(d) 不得使用計算機。

The use of calculators is not allowed.

(e) 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

注意：本屆初賽的規則有所修改，如某題的正確答案並非 0 至 9999 之間的整數，應以上述範圍內最接近正確答案的整數回答。如有兩個這樣的整數與正確答案同樣接近，則以「四捨五入」的原則取較大的整數。請細閱答題紙上的指示。

Note: There have been amendments in the regulations of the current Heat Event. If the correct answer to a question is not an integer between 0 and 9999, one should pick the integer in the above range which is closest to the correct answer. In case of an answer midway between two such integers, round up to the larger integer. Read the instructions on the answer sheet in detail.

1. 某年共有 n 個星期六。求 n 的最大可能值。 （3 分）

   There are n Saturdays in a certain year. Find the greatest possible value of n. (3 marks)

2. 若兩個正整數之積是 2012，它們之和的最小可能值是甚麼？ （3 分）

   If the product of two positive integers is 2012, what is the smallest possible value of their sum? (3 marks)

3. 現有三瓶體積相同的酒精，它們的濃度分別是 12%、34% 和 56%。若把三瓶酒精混合起來，濃度爲 x%。求 x。 （3 分）

   There are three bottles of alcohol of the same volume, whose concentrations are 12%, 34% and 56% respectively. When the three bottles are mixed together, the concentration is x%. Find x. (3 marks)

4. 有些四位正整數的千位比百位小，且十位比個位大。求這些四位數中最小的一個。 （3 分）

   Some four-digit positive integers have the property that the thousands digit is smaller than the hundreds digit, and the tens digit is larger than the unit digit. Find the smallest of these four-digit numbers. (3 marks)

5. 現有一些載重量上限爲 7 公斤的箱子。若要盛載 8 件各重 3 公斤和 5 件各重 5 公斤的物品，最少要使用幾個這樣的箱子？（每件物品不能分拆。） （4 分）

   There are some boxes which can support a maximum load of 7 kg. To carry 8 objects each with weight 3 kg and 5 objects each with weight 5 kg, what is the minimum number of such boxes needed? (Each object cannot be subdivided.) (4 marks)

6. 正整數 n 剛好有 5 個正因數。求 n 的最小可能值。（4 分）

A positive integer n has exactly 5 positive factors. Find the smallest possible value of n. (4 marks)

7. 當 2012 位數 201220122012…2012 除以 3 時，餘數是多少？（4 分）

What is the remainder when the 2012-digit number 201220122012…2012 is divided by 3? (4 marks)

8. 在一家百貨公司中，顧客購物滿 500 元即可免費成爲貴賓會員，下次購物時可享九折優惠。公司出售的貨品分爲四類，每件標價分別爲 123、246、369 和 456 元。並非貴賓會員的小冰打算購買以上四個類別的貨品各一件，那麼小冰最少要付多少元？（4 分）

In a department store, customers can become a VIP member for free if they spend 500 dollars or more. This will entitle them to a 10% discount in their next visit. There are four categories of goods and their marked prices are 123, 246, 369 and 456 dollars each respectively. Pinky, who is not a VIP member, plans to buy one item from each category. What is the lowest price (in dollars) that Pinky has to pay? (4 marks)

9. 已知 n 是四位數。若 $4n$ 的最後四位數字爲 2012，求 n 的最大可能值。（4 分）

Given n is a four-digit number. If the last four digits of $4n$ are 2012, find the greatest possible value of n. (4 marks)

10. 現有四張分別寫上「2」、「0」、「1」、「2」的卡片。若把它們重新排列，共可組成多少個不同的四位正整數？（5 分）

There are four cards, on which ‘2’, ‘0’, ‘1’, ‘2’ are printed respectively. How many different four-digit positive integers can be formed by rearranging these cards? (5 marks)

11. 設 n 爲正整數。若 12 和 n 的最小公倍數是 120，求 n 的最小可能值。（5 分）

Let n be a positive integer. If the L.C.M. of 12 and n is 120, find the smallest possible value of n. (5 marks)

12. 已知 $\sqrt{2012} \approx 44.8553$。若某個八位平方數的首四位是 2012，它的最後四位（從左至右）是甚麼？ （6 分）

Given $\sqrt{2012} \approx 44.8553$. If the first four digits of an eight-digit square number are 2012, what are the last four digits (from left to right)? (6 marks)

13. 某銳角三角形的三隻內角分別是 $a°$、$b°$ 和 $c°$，其中 a、b、c 爲正整數且 $a < b < c$。求 b 的最小可能值。 （6 分）

The three interior angles of an acute-angled triangle measure $a°$, $b°$ and $c°$ respectively, where a, b, c are positive integers such that $a < b < c$. Find the smallest possible value of b. (6 marks)

14. 已知 n 是一個三位正整數，其中任意兩個數字加起來都是偶數。問 n 有多少個不同的可能值？ （6 分）

Given n is a three-digit positive integer such that the sum of any two of its digits is even. How many different possible values of n are there? (6 marks)

15. 一項足球比賽共有 10 隊參加，其中每兩隊之間均對賽一次。每場勝方得 2 分，負方得 0 分，賽和則各得 1 分。賽事結束後，發現沒有兩隊的得分相同。那麼，得分第三高的隊伍最多有幾分？ （6 分）

There are 10 teams in a soccer tournament. Every two teams play against each other once, and the winning team scores 2 points while the losing team scores 0 point. In case of a draw, each team gets 1 point. At the end of the tournament, it was found that no two teams have the same score. What is the maximum possible score of the team with the third highest score? (6 marks)

16. 某班有 25 名學生，其中男生比女生多。已知配戴眼鏡的男生數目是沒有配戴眼鏡的女生數目的 3 倍，配戴眼鏡的女生數目是沒有配戴眼鏡的男生數目的 2 倍。求班中男生的數目。 （6 分）

There are 25 students in a class, with more boys than girls. Given that the number of boys wearing glasses is 3 times the number of girls without glasses, and that the number of girls wearing glasses is 2 times the number of boys without glasses, find the number of boys in the class. (6 marks)

17. 現要安排三名女孩和四名男孩圍著一張圓桌就坐。若任意兩名女孩都不可相鄰而坐，則安排座位的方法有多少種？（如果每人左方的人在兩種方法中皆相同，則這兩種安排座位的方法視爲相同。） （7 分）

Three girls and four boys are seated at a round table. How many different seating arrangements are there if no two girls may sit next to each other? (Two seating arrangements are regarded to be the same if every person finds the same left-hand neighbour in the two arrangements.) (7 marks)

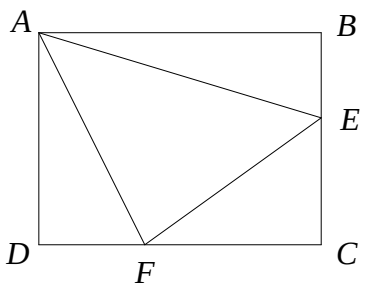
18. 在所示的乘式中，每個字母代表一個由 0 至 5 的不同數字。求 ABC 所代表的三位數的所有可能值之和。 （7 分）

In the multiplication shown, each letter represents a different digit from 0 to 5. Find the sum of all possible values of the three-digit number represented by ABC. (7 marks)

$$\begin{array}{rr} & A\ B\ C \\ \times & C\ B\ A \\ \hline D\ B\ E\ E\ B\ F & \end{array}$$

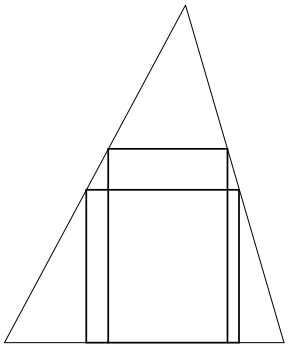
19. 圖中，*ABCD* 是長方形，面積爲 123。*E*、*F* 分別是 *BC* 和 *CD* 上的點。若 ΔABE 和 ΔADF 的面積分別是 23 和 45，求 ΔAEF 的面積。 （7 分）

In the figure, *ABCD* is a rectangle with area 123. *E* and *F* are points on *BC* and *CD* respectively. If the areas of ΔABE and ΔADF are 23 and 45 respectively, find the area of ΔAEF. (7 marks)



20. 圖中，一個邊長爲 28 的正方形和一個闊 20、高 32 的長方形內接於同一個三角形。求該三角形的面積。 （7 分）

In the figure, a square of side length 28 and a rectangle with width 20 and height 32 are inscribed in the same triangle. Find the area of the triangle. (7 marks)



再次提醒各位參賽者，作答時每題的答案均須以 0 至 9999 之間的整數表示，如有需要應以上述範圍內最接近正確答案的整數回答。請細閱答題紙上的指示。

Would contestants please be reminded again that each answer must be given in the form of an integer between 0 and 9999. Where necessary, the answer should be rounded off to the nearest integer in the above range. Read the instructions on the answer sheet in detail.

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