第十一屆培正數學邀請賽（2012 年）

11th Pui Ching Invitational Mathematics Competition (2012)

決賽（中一組）

Final Event (Secondary 1)

時限：2 小時

Time allowed: 2 hours

參賽者須知：

Instructions to Contestants:

(a) 本卷共設 20 題，總分爲 100 分。

There are 20 questions in this paper and the total score is 100.

(b) 除特別指明外，本卷內的所有數均爲十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

(c) 除特別指明外，所有答案須以數字的真確值表達，並化至最簡。不接受近似值。

Unless otherwise stated, all answers should be given in exact numerals in their simplest form. No approximation is accepted.

(d) 把所有答案填在答題紙指定的空位上。毋須呈交計算步驟。

Put your answers on the space provided on the answer sheet. You are not required to hand in your steps of working.

(e) 不得使用計算機。

The use of calculators is not allowed.

(f) 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

注意：決賽的規則與初賽不同。除特別指明外，所有答案須以數字的真確值表達，並化至最簡。不接受近似值。

Note: The rule in the Final Event is different from that in the Heat Event. Unless otherwise stated, all answers should be given in exact numerals in their simplest form. No approximation is accepted.

第 1 至第 4 題，每題 3 分。

Questions 1 to 4 each carries 3 marks.

1. 有多少個正整數 n 滿足不等式 $n^2 < 2012 < n^3$？

   How many positive integers n satisfy the inequality $n^2 < 2012 < n^3$?

2. 某校的 n 名學生從左至右排成一列。若從左邊數起的第 2012 名學生和從右邊數起的第 512 名學生是同一人，求 n 的值。

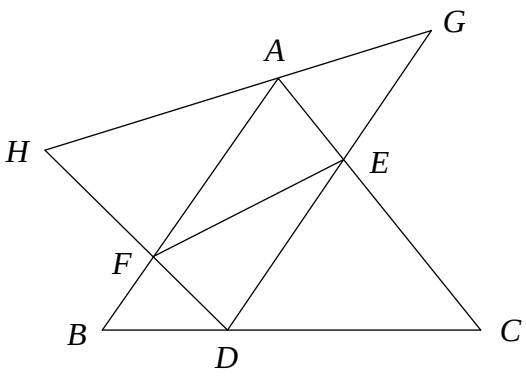
   The n students of a school are queued from left to right. If the 2012th student from the left and the 512th student from the right are the same person, find the value of n.

3. 某數學比賽設 20 道題，其中 3 分題、4 分題、5 分題、6 分題和 7 分題各佔 4 題。每題答對可得該題全部分數，否則該題得 0 分。若沛銓在比賽中得到 50 分，則他最少答對了幾題？

   There are 20 questions in a mathematical competition, with four of each of 3-mark, 4-mark, 5-mark, 6-mark and 7-mark questions. A correct answer merits all marks allocated to the question, and 0 mark is given otherwise. If Matthew gets 50 marks in the competition, what is the smallest number of questions which he answers correctly?

4. 圖中，HAG、HFD、GED、AFB、AEC 和 BDC 均爲直線，HG 與 FE 平行，且 ΔAFH 的面積爲 2012。若 $AFDE$ 爲平行四邊形，求它的面積。

   In the figure, HAG, HFD, GED, AFB, AEC and BDC are straight lines, HG is parallel to FE and the area of ΔAFH is 2012. If $AFDE$ is a parallelogram, find its area.



第 5 至第 8 題，每題 4 分。

Questions 5 to 8 each carries 4 marks.

5. 當 2012 位數 444…44 除以 37 時，餘數是多少？

   What is the remainder when the 2012-digit number 444…44 is divided by 37?

6. 一條自動行人道最初以固定速度從 *A* 向 *B* 運行，嘉希從 *A* 點沿自動行人道走往 *B* 點，共用了 30 秒，然後她沿自動行人道回到 *A* 點，再用了 60 秒。之後自動行人道的運行速度被調較至原來的兩倍，嘉希再次沿自動行人道向 *B* 點走，當她到達一半路程時，自動行人道突然停頓下來，然後嘉希繼續走向 *B* 點。假設嘉希的速度一直不變，則她第二次從 *A* 點走到 *B* 點共需多少秒？

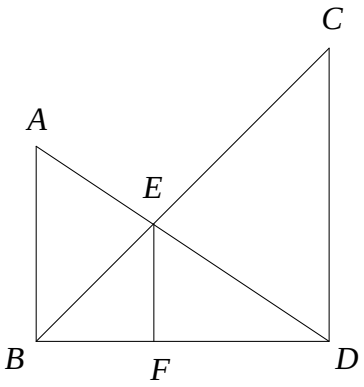
   A moving walkway was initially set to move from *A* to *B* at a constant speed. Kathy walked from *A* along the moving walkway and took 30 seconds to reach *B*. She then walked back to *A* along the moving walkway in another 60 seconds. The moving walkway was then adjusted to move at twice its original speed. Kathy walked from *A* to *B* along the moving walkway again, and when she reached halfway the moving walkway suddenly stopped and then she continued to walk towards *B*. Assuming that Kathy's speed had remained constant throughout, how many seconds would Kathy take in her second journey from *A* to *B*?

7. 對於滿足 $0<x<1$ 的實數 x，記 $x=0.\overline{a_1a_2a_3\cdots}$（當 x 是截尾小數時，我們假設之後的小數位是 0，例如 $\frac{2}{5}=0.4000\ldots$），並設 $f(x)=0.\overline{b_1b_2b_3\cdots}$，其中 $b_i=\frac{a_i+a_{i+1}}{2}$ 並下捨入至最接近整數。例如：$f(0.74747474\ldots)=0.55555555\ldots=\frac{5}{9}$（我們把 $\frac{7+4}{2}=5.5$ 下捨入至 5）。現於黑板上寫上 $\frac{1}{7}$ 這個數，一名學生每次會把黑板上的數 x 擦掉並寫上 $f(x)$ 取而代之。這個步驟重複 2012 次後，黑板上的數是甚麼？

   For a real number x satisfying $0<x<1$, denote $x=0.\overline{a_1a_2a_3\cdots}$ (when x is a terminating decimal, the subsequent decimal places are taken to be 0, e.g. $\frac{2}{5}=0.4000\ldots$), and let $f(x)=0.\overline{b_1b_2b_3\cdots}$, where $b_i=\frac{a_i+a_{i+1}}{2}$ rounded down to the nearest integer. For example, $f(0.74747474\ldots)=0.55555555\ldots=\frac{5}{9}$ ($\frac{7+4}{2}=5.5$ is rounded down to 5). Now the number $\frac{1}{7}$ is written onto a blackboard. Each time a student will erase the number x on the blackboard and then write down the number $f(x)$ to replace it. After repeating this process 2012 times, what will be the number on the blackboard?

8. 圖中，*AED*、*BEC* 和 *BFD* 均爲直線，且 *AB*、*CD* 和 *EF* 均與 *BD* 垂直。若 $AB=7$、$EF=6$ 和 $CE=60$，求 *BF*。

In the figure, *AED*, *BEC* and *BFD* are straight lines, while *AB*, *CD* and *EF* are all perpendicular to *BD*. If $AB=7$, $EF=6$ and $CE=60$, find *BF*.



第 9 至第 12 題，每題 5 分。

Questions 9 to 12 each carries 5 marks.

9. 設 *n* 爲正整數，它有超過一個質因數，而各質因數之和爲 *s*。若 *n* 是 *s* 的倍數，求 *n* 的最小可能值。

Let *n* be a positive integer with more than one prime factor. The sum of the prime factors of *n* is equal to *s*. If *n* is a multiple of *s*, find the smallest possible value of *n*.

10. 方程 $xyz=1400$ 有多少組正整數解？

How many sets of positive integer solutions are there to the equation $xyz=1400$?

11. 一個八位正整數 *n* 由數字 1、2、…、8 組成（沒有重複），且 *n* 可被 99 整除。求 *n* 的最大可能值。

An eight-digit positive integer *n* is made up of the digits 1, 2, …, 8 without repetition, and is divisible by 99. What is the largest possible value of *n*?

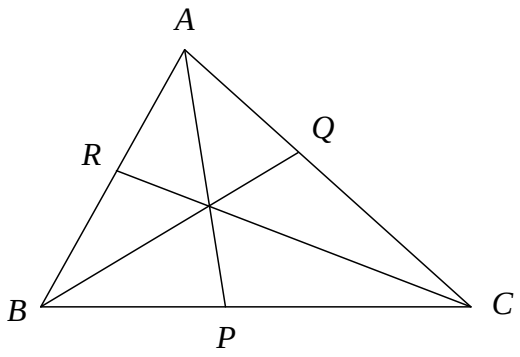
12. 現把 8 張分別寫上 1 至 8 的卡紙隨意分發給小美和小麗，每人獲發 4 張。兩人各自計算自己的卡紙上各數之和及各數之積。若小美所得的和較小麗小，而小美所得的積較小麗大，求小美所得的積的最大可能值。

There are 8 cards numbered 1 to 8, and they are randomly distributed to Annie and Betty so that each has 4 cards. Each of them computes the sum and product of the numbers on their cards. If the sum obtained by Annie is smaller than that obtained by Betty while the product obtained by Annie is larger than that obtained by Betty, find the greatest possible value of the product obtained by Annie.

第 13 至第 16 題，每題 6 分。

Questions 13 to 16 each carries 6 marks.

13. 圖中，P 是 BC 的中點，Q、R 則分別是 CA 和 AB 上的點，使得 AP、BQ 和 CR 三線共點。若 $BP=21$、$BR=30$、$RA=15$ 且 $AQ=13$，求 ΔAQR 的面積。

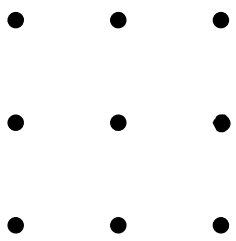


In the figure, P is the mid-point of BC. Q and R are points on CA and AB respectively such that the lines AP, BQ and CR are concurrent. If $BP=21$, $BR=30$, $RA=15$ and $AQ=13$, find the area of ΔAQR.

14. $ABCDEFGH$ 是個正方體。一隻螞蟻可選其中一個頂點出發，沿正方體的棱爬到對角的頂點，當中必須途經所有頂點，而且不能經過同一點超過一次。那麼螞蟻爬行的路線有多少個不同的選擇？

ABCDEFGH is a cube. An ant may choose to start from one of the vertices and crawls along the edges of the cube to the opposite vertex, subject to the conditions that all vertices must be traversed and no point may be visited more than once. How many different choices are there for the route of the ant?

15. 如圖所示，九點排成 3×3 的陣列。現要從中選取四點，使得當中沒有三點成一直線，問共有多少種不同的選法？



The figure shows nine points arranged in a 3×3 array. Now four points are to be chosen so that no three chosen points lie on the same straight line. How many different choices are there?

16. 在一個 7×7 方格表的每格中均有一個正整數。已知整個表格中共有 k 個不同的正整數，且當兩格有公共頂點時，該兩格內的兩數之和均爲質數。求 k 的最大可能值。

In each cell of a 7×7 table there is a positive integer. It is known that there are altogether k different positive integers in the entire table, and that whenever two cells share a common vertex, the sum of their two numbers must be prime. Find the greatest possible value of k.

第 17 至第 20 題，每題 7 分。

Questions 17 to 20 each carries 7 marks.

17. 老師寫下了一個五位數 N，然後讓小陳看 N 的首三位數字、讓小李看 N 中間的三位數字、讓小張看 N 的最後三位數字。之後老師要求每人寫下一些他們知道關於 N 的性質。

    小陳寫道：「N 不是 101 的倍數。」
    小李寫道：「N 中間的三位數字之和是奇數。」
    小張寫道：「N 不是 91 的倍數。」

    求 N 的值。

    The teacher wrote down a five-digit number N and then let Alan see the first three digits of N, let Bob see the three digits of N in the middle and let Carl see the last three digits of N. The teacher subsequently asked each person to write down something they knew about N.

    Alan wrote, 'N is not a multiple of 101'.
    Bob wrote, 'the sum of the three digits of N in the middle is odd.'
    Carl wrote, 'N is not a multiple of 91'.

    Find the value of N.

18. 設 m 爲方程 $3x+2y+z=2008$ 正整數解的數目，n 爲方程 $3x+2y+z=2012$ 正整數解的數目。已知 m 的最後三位數字爲 002，求 n 的最後三位數字。

    Let m be the number of positive integer solutions to the equation $3x+2y+z=2008$, and n be the number of positive integer solutions to the equation $3x+2y+z=2012$. Given that the last three digits of m are 002, find the last three digits of n.

19. 在所示的算式中，每個字母代表一個由 0 至 9 的不同數字，且沒有字母代表的數字是 7。若 MATHS 所代表的五位數爲質數，求此質數。

    In the multiplication shown, each letter represents a different digit from 0 to 9 and no letter represents 7. If the five-digit number represented by MATHS is prime, find this prime number.

$$\begin{array}{rr} & \text{M A T H S} \\ \times & \qquad\qquad 7 \\ \hline & \text{P O I S O N} \end{array}$$

20. 有多少個四位正整數的數字之積是正平方數？

How many four-digit positive integers have their product of digits being a positive square number?

全卷完

END OF PAPER