

第九屆培正數學邀請賽
9th Pui Ching Invitational Mathematics Competition

決賽（中一組）
Final Event (Secondary 1)

時限：2 小時

Time allowed: 2 hours

參賽者須知：

Instructions to Contestants:

- (a) 本卷共設 20 題，總分爲 100 分。

There are 20 questions in this paper and the total score is 100.

- (b) 除特別指明外，本卷內的所有數均爲十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

- (c) 除特別指明外，所有答案須以數字的真確值表達，並化至最簡。不接受近似值。

Unless otherwise stated, all answers should be given in exact numerals in their simplest form.
No approximation is accepted.

- (d) 把所有答案填在答題紙指定的空位上。毋須呈交計算步驟。

Put your answers on the space provided on the answer sheet. You are not required to hand in your steps of working.

- (e) 不得使用計算機。

The use of calculators is not allowed.

- (f) 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

第 1 至第 4 題，每題 3 分。

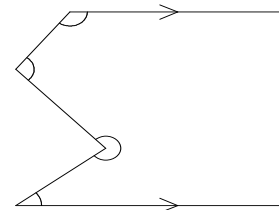
Questions 1 to 4 each carries 3 marks.

1. 求 2010×320 的最後四位數字。

Find the last four digits of 2010×320 .

2. 圖中畫有標記的四個角的總和是 x° 。求 x 。

In the figure, the sum of the four marked angles is x° . Find x .



3. 設 $[x]$ 代表不超過 x 的最大整數，例如 $[1.1] = 1$ 、 $[6.9] = 6$ 和 $[5] = 5$ 。求 $\left\lfloor \frac{1004 \times 1997}{2010} \right\rfloor + \left\lfloor \frac{1006 \times 1997}{2010} \right\rfloor$ 的值。

Let $[x]$ denote the greatest integer not exceeding x . For example, $[1.1] = 1$, $[6.9] = 6$ and $[5] = 5$.

Find the value of $\left\lfloor \frac{1004 \times 1997}{2010} \right\rfloor + \left\lfloor \frac{1006 \times 1997}{2010} \right\rfloor$.

4. 一家百貨店進行「買二送一」推廣，顧客每次購買兩件貨品，可獲贈一件貨品，但贈品的價值須分別不高於購買的兩件貨品的價值。美詩希望得到 15 件價值分別是 1 元、2 元、 \dots 、15 元的貨品，則她最少要付多少元？

A department store has launched a 'buy 2 get 1 free' promotion. Each time when one buys 2 articles, one is entitled to get an article for free, subject to the condition that the value of the free article must not exceed each of the two purchased articles. If Macy wants to get 15 articles valued at 1 dollar, 2 dollars, ..., 15 dollars respectively, what is the minimum amount (in dollars) that she has to pay?

第 5 至第 8 題，每題 4 分。

Questions 5 to 8 each carries 4 marks.

5. 設 $n = 101001^6 \times 582002^2 \div 52007$ 。已知 n 是整數，求 n 的最後三位數字。

Let $n = 101001^6 \times 582002^2 \div 52007$. It is known that n is an integer. Find the last three digits of n .

6. 在所示的乘式中，每個字母代表一個由 0 至 9 的不同數字。
求乘積 BCADEEE 所代表的七位數。

In the multiplication shown, each letter represents a different digit from 0 to 9. Find the seven-digit number represented by the product BCADEEE.

$$\begin{array}{r} \\ \\ \times \\ \hline B \ C \ A \ D \ E \ E \ E \end{array}$$

7. 某班有 9 名學生，學號分別是 1 至 9。若要把他們分成 3 組，每組 3 人，使得每組的學生的學號之和都是 9 的倍數，共有多少種分組方法？

In a class there are 9 students, numbered 1 to 9. In how many different ways can the students be divided into 3 groups of 3 students each such that the sum of the numbers of the students in each group is divisible by 9?

8. 某五位數 n 的五個數字分別是 1、3、4、5 和 6。已知 n 是 11 的倍數，求 n 的最小可能值。

The five digits of a five-digit number n are 1, 3, 4, 5 and 6. Given that n is a multiple of 11, find the smallest possible value of n .

第 9 至第 12 題，每題 5 分。

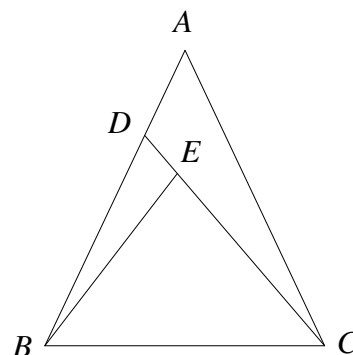
Questions 9 to 12 each carries 5 marks.

9. 若要從首 2010 個正整數中選兩個不同的數，使得它們的最大公因數可被 30 整除，問有多少個不同的選擇？

If two different numbers are to be chosen from the first 2010 positive integers such that their H.C.F. is divisible by 30, how many different choices are there?

10. 圖中， E 是 $\triangle ABC$ 內的一點， CE 延長後交 AB 於 D 。若 $AB = 6$ 、 $BC = 5$ 、 $CA = 7$ 且 $\triangle ABC \sim \triangle CEB$ ，求 $CD \times CE$ 。

In the figure, E is a point inside $\triangle ABC$ and CE is produced to meet AB at D . If $AB = 6$, $BC = 5$, $CA = 7$ and $\triangle ABC \sim \triangle CEB$, find $CD \times CE$.



11. 兩名運動員在圓形的跑步徑上分別以均速向相同的方向跑步。他們同時在 O 點出發，之後不斷在跑步徑上相遇。設他們第 1 次、第 2 次和第 5 次相遇的地方分別為 A 、 B 和 E 。若沿跑步徑量度， O 和 A 的最短距離是 100 米，而 E 和 B 的最短距離則是 230 米。求跑步徑的長度（以米為單位）。

Two athletes ran on a circular jogging track in the same direction, each at uniform speed. They started from point O at the same time and then met on the track repeatedly. Let their first, second and fifth meeting points be A , B and E respectively. When measured along the track, the shortest distance between O and A is 100 m while the shortest distance between E and B is 230 m. Find the length of the track (in m).

12. 設 $[x]$ 代表不超過 x 的最大整數，且 $\{x\} = x - [x]$ ，例如 $[1.1] = 1$ 、 $\{6.9\} = 0.9$ 和 $[5] = 5$ 。若 x 是正數，且 $2[x]\{x\} + 4 = 3\{x\} + 3x$ ，求 x 所有可能值之和。

Let $[x]$ denote the greatest integer not exceeding x and $\{x\} = x - [x]$. For example, $[1.1] = 1$, $\{6.9\} = 0.9$ and $[5] = 5$. If x is a positive number such that $2[x]\{x\} + 4 = 3\{x\} + 3x$, find the sum of all possible values of x .

第 13 至第 16 題，每題 6 分。

Questions 13 to 16 each carries 6 marks.

13. 求 $\frac{1}{10} + \frac{1}{100} + \frac{2}{1000} + \frac{3}{10000} + \frac{5}{100000} + \dots$ 的值。（各項的分子是數列 1, 1, 2, 3, 5, 8, 13, ... 的連續項，此數列的首兩項為 1，之後每項都是前兩項之和。）

Find the value of $\frac{1}{10} + \frac{1}{100} + \frac{2}{1000} + \frac{3}{10000} + \frac{5}{100000} + \dots$. (The numerators of the summands are consecutive terms of the sequence 1, 1, 2, 3, 5, 8, 13, ..., in which the first two terms are 1 and each subsequent term is the sum of the two preceding terms.)

14. 美芬想了四個正整數 a 、 b 、 c 和 d ，其中 $a > b > c > d$ 。她求得 $a - b$ 、 $a - c$ 、 $a - d$ 、 $b - c$ 、 $b - d$ 和 $c - d$ 的值後，發現其中四個是 7、9、14 和 16。求 $a - d$ 所有可能值之和。

Ivy thought of four positive integers a , b , c and d where $a > b > c > d$. After computing the values of $a - b$, $a - c$, $a - d$, $b - c$, $b - d$ and $c - d$, she found that four of these values were 7, 9, 14 and 16. Find the sum of all possible values of $a - d$.

15. 設 $g(x) = \frac{x}{4} + 4 + \sqrt{x-4}$ 。求 $\underbrace{g(g(g(\cdots g(13)\cdots)))}_{10 \text{ 個 } g}$ 的值，答案準確至小數點後 3 位。

Let $g(x) = \frac{x}{4} + 4 + \sqrt{x-4}$. Find the value of $\underbrace{g(g(g(\cdots g(13)\cdots)))}_{10 \text{ copies of } g}$ correct to 3 decimal places.

16. 已知 n 是正整數，它被 7 除時餘 3，被 11 除時餘 5，被 13 除時餘 6，被 101 除時餘 50。求 n 的最小可能值。

Let n be a positive integer which leaves a remainder of 3 when divided by 7, a remainder of 5 when divided by 11, a remainder of 6 when divided by 13 and a remainder of 50 when divided by 101. Find the smallest possible value of n .

第 17 至第 20 題，每題 7 分。

Questions 17 to 20 each carries 7 marks.

17. 設 x, y, z 為正整數，其中 $x < y < z$ 且 $x + y + z = 155$ 。敏儀計算了 x 和 y 的最大公因數、 x 和 z 的最大公因數及 y 和 z 的最大公因數，並發現這三個最大公因數當中，其中一個等於另外兩個之積。求 x 所有可能值之和。

Let x, y, z be positive integers such that $x < y < z$ and $x + y + z = 155$. Mandy computes the H.C.F. of x and y , the H.C.F. of x and z as well as the H.C.F. of y and z . She finds that among these three H.C.F.s, one is equal to the product of the other two. Find the sum of all possible values of x .

18. 一個袋子中共有 100 枚硬幣，其中每個都是二元、五元或十元硬幣。若這 100 個硬幣共值 n 元，問 n 有多少個不同的可能值？

There are 100 coins in a bag, each of denomination 2 dollars, 5 dollars or 10 dollars. If the total value of the 100 coins is n dollars, how many different possible values of n are there?

19. 在課室裡，老師對五名學生小陳、小李、小張、小王和小何說：「我寫下了一個五位數 N ，它由五個不同的數字組成。我會讓小陳看 N 的萬位和千位，讓小李看千位和百位，讓小張看百位和十位，讓小王看十位和個位，並讓小何看個位和萬位。」之後老師如所述般讓每名學生知道 N 的兩個數字，然後各人圍圈而坐，展開了以下的對話。

老師說：「知道 N 的一個質因數的請舉手」，之後有兩名學生舉手。

老師再問：「知道 N 的一個質因數的請舉手」，這次有三名學生舉手。

老師說：「知道 N 的一個合成數因數的請舉手」，之後有兩名學生舉手。

老師又說：「知道 N 的兩個合成數因數的請舉手」，但沒有學生舉手。

之後老師問：「誰知道 N 的值？」

其中一名學生說：「我知道， N 是 59 的倍數呢。」

假設所有學生都是聰明的（即有足夠資料便能作出推論），求 N 。

In a classroom, the teacher said to five students, Alan, Bob, Carl, Dick and Eason, 'I have written down a five-digit number N which is made up of five different digits. I will let Alan see the ten thousands and thousands digits of N , let Bob see the thousands and hundreds digits, let Carl see the hundreds and tens digits, let Dick see the tens and unit digits and let Eason see the unit and ten thousands digits.' The teacher then let each student know two digits of N as said, and then everybody sat in a circle and started the following conversation.

'Raise your hands if you know a prime factor of N ,' said the teacher, and then two students raised their hands.

'Raise your hands if you know a prime factor of N ,' asked the teacher again, and this time three students raised their hands.

'Raise your hands if you know a composite factor of N ,' the teacher continued, and then two students raised their hands.

'Raise your hands if you know two composite factors of N ,' said the teacher, but no student raised their hands.

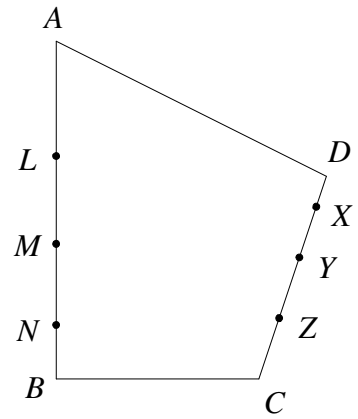
Then the teacher asked, 'who knows the value of N ?'

One student said, 'I know. N is a multiple of 59.'

Assuming all students to be clever (which means that deductions can be made whenever sufficient information is given), find N .

20. 圖中，四邊形 $ABCD$ 的面積為 180。 L 、 M 、 N 是 AB 上的點，使得 $AL:LM:MN:NB=4:3:2:1$ ； X 、 Y 、 Z 則是 DC 上的點，使得 $DX:XY:YZ:ZC=2:4:7:7$ 。若 $\triangle AXD$ 和四邊形 $AMYD$ 的面積分別是 12 和 78，求四邊形 $LNZX$ 的面積。

In the figure, $ABCD$ is a quadrilateral with area 180. L , M , N are points on AB such that $AL:LM:MN:NB=4:3:2:1$, while X , Y , Z are points on DC such that $DX:XY:YZ:ZC=2:4:7:7$. If the areas of $\triangle AXD$ and quadrilateral $AMYD$ are 12 and 78 respectively, find the area of quadrilateral $LNZX$.



全卷完

END OF PAPER