第八屆培正數學邀請賽

8th Pui Ching Invitational Mathematics Competition

初賽（高中組）

Heat Event (Senior Secondary)

時限：1 小時 15 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

1. 本卷共設 20 題，總分爲 100 分。

   There are 20 questions in this paper and the total score is 100.

2. 除特別指明外，本卷內的所有數均爲十進制。

   Unless otherwise stated, all numbers in this paper are in decimal system.

3. 所有答案皆是 0 至 9999 之間的整數（包括 0 和 9999）。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

   All answers are integers between 0 and 9999 (including 0 and 9999). Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

4. 不得使用計算機。

   The use of calculators is not allowed.

5. 本卷的附圖不一定依比例繪成。

   The diagrams in this paper are not necessarily drawn to scale.

1. 現於一個六邊形的每個頂點寫上一個正整數，使得相鄰頂點上所寫的整數不同。求六個頂點上的正整數之和的最小可能值。（3 分）

A positive integer is to be assigned to each vertex of a hexagon. If it is required that integers assigned to adjacent vertices be different, find the smallest possible value of the sum of the six integers at the six vertices. (3 marks)

2. 若 x 和 y 均爲三位數而 $2x=3y$，求 x 和 y 之差的最大可能值。（3 分）

If x and y are three-digit numbers such that $2x=3y$, find the greatest possible difference between x and y. (3 marks)

3. 若方程 $f(x)=120$ 的其中一個解是 $x=60$，則哪個數必定是方程 $f(2x)=120$ 的解？（3 分）

If the equation $f(x)=120$ has a solution $x=60$, which number must be a solution to the equation $f(2x)=120$? (3 marks)

4. 求 $\dfrac{3^4+6^4+9^4+\cdots+3000^4}{1^4+2^4+3^4+\cdots+1000^4}$ 的值。（3 分）

Find the value of $\dfrac{3^4+6^4+9^4+\cdots+3000^4}{1^4+2^4+3^4+\cdots+1000^4}$. (3 marks)

5. 求最接近 $3+\dfrac{3}{\pi}+\dfrac{3}{\pi^2}+\dfrac{3}{\pi^3}+\cdots$ 的整數。（4 分）

Find the integer closest to $3+\dfrac{3}{\pi}+\dfrac{3}{\pi^2}+\dfrac{3}{\pi^3}+\cdots$. (4 marks)

6. 設 $p(x)$ 爲 n 次多項式。若 $\lim\limits_{x\to\infty}\dfrac{1+2x^{345}+6x^{789}}{p(x)}=2009$，求 n。（4 分）

Let $p(x)$ be a polynomial of degree n. If $\lim\limits_{x\to\infty}\dfrac{1+2x^{345}+6x^{789}}{p(x)}=2009$, find n. (4 marks)

7. 若 $\int_0^{\frac{\pi}{2}} f(\sin x)\,\mathrm{d}x=60$，求 $\int_0^{\pi} f(\sin x)\,\mathrm{d}x$ 的值。（4 分）

If $\int_0^{\frac{\pi}{2}} f(\sin x)\,\mathrm{d}x=60$, evaluate $\int_0^{\pi} f(\sin x)\,\mathrm{d}x$. (4 marks)

8. 設 p 為質數。方程 $x^3+2px^2+px=0$ 有多少個不同的實根？ （4分）

Let p be a prime number. How many different real roots to the equation $x^3+2px^2+px=0$ are there? (4 marks)

9. 一個盒子中有 2009 個球，分別編號為 1 至 2009。最少要從盒子中抽多少個球，才能保證其中兩個抽出的球的號碼是兩個連續數？ （4分）

In a box there are 2009 balls numbered 1 to 2009. What is the minimum number of balls drawn to ensure that two balls with consecutive numbers are drawn? (4 marks)

10. 某校有相同數目的男生和女生。在一次測驗中，男生和女生的平均得分相同，標準差則分別是 10 分和 20 分。若以全校學生計算，測驗得分的標準差是 n 分。求最接近 n 的整數。 （5分）

In a school there is an equal number of boys and girls. In a test, the mean scores for boys and girls are the same, while the standard deviations are 10 marks and 20 marks respectively. The overall standard deviation of the test score is n marks. Find n correct to the nearest integer. (5 marks)

11. 有多少個三位正整數的十位比個位和百位都大？ （5分）

How many three-digit positive integers have their tens digit greater than both the unit digit and the hundreds digit? (5 marks)

12. 方程 $\sin 2x° = \sin 2009°$ 在 $0 < x < 360$ 區間內有多少個解？ （5分）

How many solutions are there to the equation $\sin 2x° = \sin 2009°$ in the interval $0 < x < 360$? (5 marks)

13. 某等差數列的每項皆是整數。若它的首項是 1，第 n 項是 2009，求 n 所有可能值之和。 （6分）

In an arithmetic sequence, each term is an integer. If the first term is 1 and the n-th term is 2009, find the sum of all possible values of n. (6 marks)

14. 設 n 爲正整數。在二進制中，$9n$ 是個九位數，其中五個位是「1」。求 n 的最小可能值。（6分）

Let n be a positive integer. In binary notation, $9n$ has 9 digits, five of which are '1'. Find the smallest possible value of n. (6 marks)

15. 已知 $p(x)$ 是多項式，而它的所有系數都是正整數。設 $p'''(x)$ 爲 $p(x)$ 的三次導數。若 $p'''(0)$ 是兩位正整數，那麼 $p(x)$ 裏 x^3 的系數有多少個不同的可能值？（6分）

It is known that $p(x)$ is a polynomial with positive integer coefficients. Let $p'''(x)$ be the third derivative of $p(x)$. If $p'''(0)$ is a two-digit positive integer, how many different possible values are there for the coefficient of x^3 in $p(x)$? (6 marks)

16. 設 n 和 r 爲正整數。若 C_r^n 可被 2009 整除，求 n 的最小可能值。（7分）

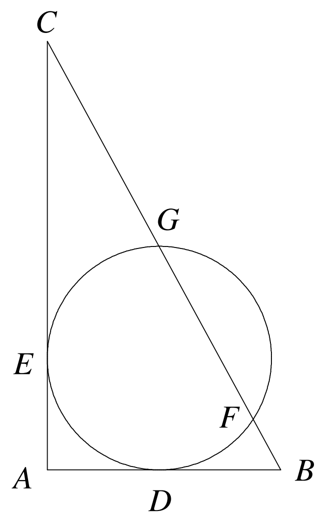
Let n and r be positive integers. If C_r^n is divisible by 2009, find the smallest possible value of n. (7 marks)

17. 已知 $0.30102 < \log 2 < 0.30103$，且 n 爲正整數。當 n 寫成二進制時，它是一個 75 位數；當 n 寫成十進制時，它是一個多少位的數？（7分）

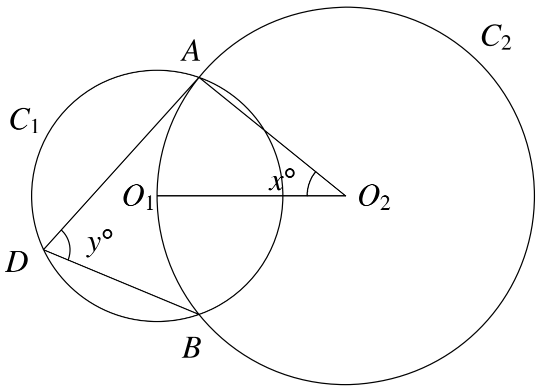
Given $0.30102 < \log 2 < 0.30103$ and n is a positive integer. In binary representation, n is a 75-digit number. How many digits does n have in decimal notation? (7 marks)

18. 圖中，ABC 是直角三角形，A 是直角。AB 和 AC 分別切圓於 D 和 E，BC 與圓相交於 F 和 G。若 $AE = 8$、$EC = 15$、$CG = 9$，且以最簡分數表示時 $AB = \frac{m}{n}$，求 $m+n$。（7分）

In the figure, ΔABC is right-angled at A. AB and AC are tangent to the circle at D and E respectively, and BC meets the circle at F and G. If $AE = 8$, $EC = 15$ and $CG = 9$, and that $AB = \frac{m}{n}$ in lowest term, find $m+n$. (7 marks)



19. 圖中，O_1 和 O_2 分別是 C_1 和 C_2 的圓心，其中 C_1 比 C_2 小，且 C_2 穿過 O_1。兩圓相交於 A 和 B，而 D 是 C_1 的圓周上並位於 C_2 外的一點。已知 $\angle AO_2O_1 = x°$、$\angle ADB = y°$，其中 x、y 都是兩位數，且 y 可經由把 x 的數字左右倒轉而得到。求 x。（7 分）



In the figure, O_1 and O_2 are the centres of circles C_1 and C_2 respectively, with C_1 smaller than C_2 and C_2 passing through O_1. The two circles meet at A and B, and D is a point on the circumference of C_1 outside C_2. Suppose $\angle AO_2O_1 = x°$ and $\angle ADB = y°$, where x, y are both two-digit numbers with y formed by reversing the digits of x. Find x. (7 marks)

20. 在所示的加法中，每個字母代表一個 0 至 9 的不同數字。求 TWO 所代表的三位數的所有可能值之和。（7 分）

In the addition shown, each letter represents a different digit from 0 to 9. Find the sum of all possible values of the three-digit number represented by TWO. (7 marks)

$$\begin{array}{r} T\ W\ O \\ +\quad T\ W\ O \\ \hline F\ O\ U\ R \end{array}$$

全卷完

END OF PAPER