第八屆培正數學邀請賽

8th Pui Ching Invitational Mathematics Competition

初賽（中四組）

Heat Event (Secondary 4)

時限：1 小時 15 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

1. 本卷共設 20 題，總分爲 100 分。

   There are 20 questions in this paper and the total score is 100.

2. 除特別指明外，本卷內的所有數均爲十進制。

   Unless otherwise stated, all numbers in this paper are in decimal system.

3. 所有答案皆是 0 至 9999 之間的整數（包括 0 和 9999）。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

   All answers are integers between 0 and 9999 (including 0 and 9999). Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

4. 不得使用計算機。

   The use of calculators is not allowed.

5. 本卷的附圖不一定依比例繪成。

   The diagrams in this paper are not necessarily drawn to scale.

1. 如果某數由左至右和由右至左看皆相同，我們稱這個數是「回文數」。例如 3883、12321 和 25052 都是「回文數」。若 n 是大於 2009 的「回文數」，求 n 的最小可能值。（3 分）

If a number reads the same from left to right as from right to left, it is called a 'palindrome'. For example, 3883, 12321 and 25052 are 'palindromes'. If n is a 'palindrome' greater than 2009, find the smallest possible value of n. (3 marks)

2. 一個 n 邊形的頂點數目和邊的數目之和爲 2000。求 n。（3 分）

The sum of the number of vertices and the number of edges of an n-sided polygon is 2000. Find n. (3 marks)

3. 設 p 爲質數。方程 $x^2+2px+p=0$ 有多少個不同的實根？（3 分）

Let p be a prime number. How many different real roots to the equation $x^2+2px+p=0$ are there? (3 marks)

4. 求最大的四位數 $\overline{abcd}$，使得兩位數 $\overline{ab}$、$\overline{bc}$ 和 $\overline{cd}$ 皆是平方數。（3 分）

Find the largest four-digit number $\overline{abcd}$ for which the two-digit numbers $\overline{ab}$, $\overline{bc}$ and $\overline{cd}$ are all square numbers. (3 marks)

5. 若某正整數除以 2009 時的餘數是 1234，則它除以 7 時的餘數是多少？（4 分）

If a positive integer leaves a remainder of 1234 when divided by 2009, what is the remainder when it is divided by 7? (4 marks)

6. 設 n 爲 2009 位數「2009000…0002009」。求 n 的最小質因數。（4 分）

Let n be the 2009-digit number '2009000…0002009'. Find the smallest prime factor of n. (4 marks)

7. 小美跟小婷和小妮合作，用了 1 天便完成了一件工作。若小美自己進行，則需 2009 天才能完成同一件工作。已知小婷和小妮各自的工作效率均爲小美的 n 倍，求 n。（4 分）

Amy completes a project with Betty and Bonnie in 1 day. If Amy works on her own, it would take 2009 days to complete the same project. It is known that the working efficiency of each of Betty and Bonnie is n times that of Amy. Find n. (4 marks)

8. 某平方數的個位數字是 5。求它的十位數字。 （4 分）

The unit digit of a square number is 5. Find its tens digit. (4 marks)

9. 在一個遊戲中有 100 個盒子，每個盒子內均有一個紅色或藍色的球。在每個回合，參加者可選兩個盒子並把它們同時打開。如果兩個盒子的球的顏色相同，則參加者可獲獎，否則把盒子關上然後進行下一回合。遊戲最少要進行多少個回合參加者才可保證獲獎？ （4 分）

In a game, there are 100 boxes, each of which contains a red or blue ball. In each round the player chooses two boxes and opens them at the same time. If they contain two balls of the same colour, the player wins a prize, otherwise the boxes are closed and the player proceeds to the next round. What is the minimum number of rounds needed to ensure a prize to be won? (4 marks)

10. 方程 $\sin 2x° = \sin 2009°$ 在 $0 < x < 360$ 區間內有多少個解？ （5 分）

How many solutions are there to the equation $\sin 2x° = \sin 2009°$ in the interval $0 < x < 360$? (5 marks)

11. 在 $(x^{1000}+1)^3(x+1)^{10}$ 的展開式中，x^{2009} 的系數是多少？ （5 分）

What is the coefficient of x^{2009} in the expansion of $(x^{1000}+1)^3(x+1)^{10}$? (5 marks)

12. 一隻猴子在第一天吃 1 個蘋果，之後每一天都吃之前一天所吃的蘋果的 r 倍，其中 r 是正整數。已知該猴子在某天吃了 4096 個蘋果，求 r 所有可能值之和。 （6 分）

A monkey eats 1 apple on the first day. On each subsequent day, it eats r times as many apples as on the previous day, where r is a positive integer. Given that the monkey eats 4096 apples on a certain day, find the sum of all possible values of r. (6 marks)

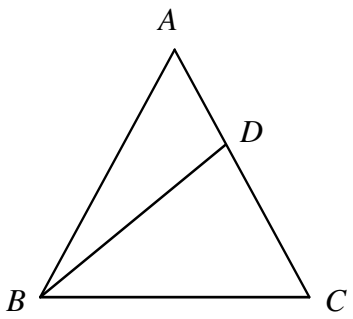
13. 現要安排 10 名編號 1 至 10 的小朋友坐在 1 至 10 號座位。若當中必須有剛好 7 名小朋友坐在自己的編號的座位，則座位的編排有多少種不同的方法？ （6 分）

10 children numbered 1 to 10 are to be arranged to 10 seats numbered 1 to 10. If it is required that exactly 7 children be assigned to a seat matching their numbers, how many different seatings are possible? (6 marks)

14. 設 n 爲正整數。若 $\sqrt{n^2-21n+114}$ 是質數，求 n 所有可能值之和。（6分）

Let n be a positive integer. If $\sqrt{n^2-21n+114}$ is a prime number, find the sum of all possible values of n. (6 marks)

15. 圖中，$AB = AC$，而 D 是 B 到 AC 的垂足。若 $AD:DC=5:8$ 而 $BD = 10$，求 ΔABC 的面積，答案準確至最接近整數。（6分）



In the figure, $AB = AC$ and D is the foot of the perpendicular from B to AC. If $AD:DC=5:8$ and $BD = 10$, find the area of ΔABC correct to the nearest integer. (6 marks)

16. 現有兩個 12 小時制的時鐘。某天正午，兩個鐘都顯示著正確的時間，但第一個鐘每小時走快 12 分鐘，第二個鐘每小時走慢 15 分鐘。小明在之後的每小時都會看看兩個鐘的時間。在多少小時後小明才會首次看到兩個鐘顯示相同的時間？（6分）

There are two 12-hour clocks. At noon on a certain day, both clocks are showing the correct time. However, in each subsequent hour the first clock runs 12 minutes fast while the second clock runs 15 minutes slow. Mike checks the two clocks in every subsequent hour. After how many hours will he first see the two clocks showing the same time? (6 marks)

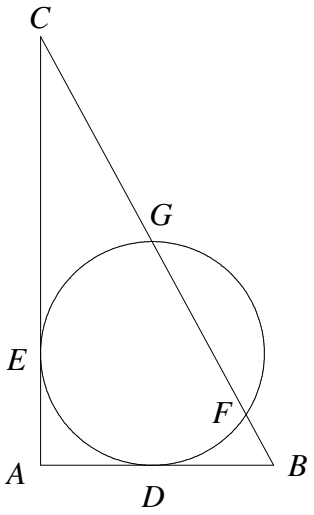
17. 已知 n 是三位正整數，且剛好有三個正因數。若把 n 的數字左右倒轉，所得的三位數也剛好有三個正因數。求 n 所有可能值之和。（7分）

Given n is a three-digit positive integer with exactly three positive factors. If the three digits of n are reversed, the new three-digit number formed also has exactly three positive factors. Find the sum of all possible values of n. (7 marks)

18. 已知 $0.30102 < \log 2 < 0.30103$，且 n 爲正整數。若 2^n 是一個 20 位數，且其個位數字是 2，求 n。（7分）

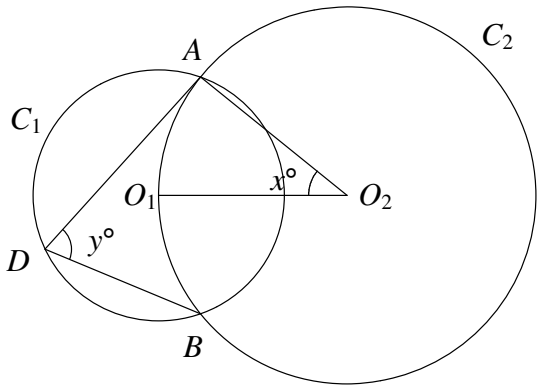
Given $0.30102 < \log 2 < 0.30103$ and n is a positive integer. If 2^n is a 20-digit number with unit digit 2, find n. (7 marks)

19. 圖中，ABC 是直角三角形，A 是直角。AB 和 AC 分別切圓於 D 和 E，BC 與圓相交於 F 和 G。若 $AE = 8$、$EC = 15$、$CG = 9$，且以最簡分數表示時 $AB = \frac{m}{n}$，求 $m+n$。（7 分）



In the figure, ΔABC is right-angled at A. AB and AC are tangent to the circle at D and E respectively, and BC meets the circle at F and G. If $AE = 8$, $EC = 15$ and $CG = 9$, and that $AB = \frac{m}{n}$ in lowest term, find $m+n$. (7 marks)

20. 圖中，O_1 和 O_2 分別是 C_1 和 C_2 的圓心，其中 C_1 比 C_2 小，且 C_2 穿過 O_1。兩圓相交於 A 和 B，而 D 是 C_1 的圓周上並位於 C_2 外的一點。已知 $\angle AO_2O_1 = x°$、$\angle ADB = y°$，其中 x、y 都是兩位數，且 y 可經由把 x 的數字左右倒轉而得到。求 x。（7 分）



In the figure, O_1 and O_2 are the centres of circles C_1 and C_2 respectively, with C_1 smaller than C_2 and C_2 passing through O_1. The two circles meet at A and B, and D is a point on the circumference of C_1 outside C_2. Suppose $\angle AO_2O_1 = x°$ and $\angle ADB = y°$, where x, y are both two-digit numbers with y formed by reversing the digits of x. Find x. (7 marks)

全卷完

END OF PAPER