

第八屆培正數學邀請賽

8th Pui Ching Invitational Mathematics Competition

初賽（中三組）

Heat Event (Secondary 3)

時限：1 小時 15 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

1. 本卷共設 20 題，總分爲 100 分。

There are 20 questions in this paper and the total score is 100.

2. 除特別指明外，本卷內的所有數均爲十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

3. 所有答案皆是 0 至 9999 之間的整數（包括 0 和 9999）。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

All answers are integers between 0 and 9999 (including 0 and 9999). Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

4. 不得使用計算機。

The use of calculators is not allowed.

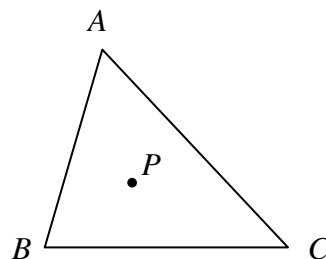
5. 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

1. 若 $x - 2009 = y + 1234$ ，求 x 和 y 兩數之差。 (2 分)
If $x - 2009 = y + 1234$, find the difference between the two numbers x and y . (2 marks)
2. 如果某數由左至右和由右至左看皆相同，我們稱這個數是「回文數」。例如 3883、12321 和 25052 都是「回文數」。求最小的四位「回文數」。 (3 分)
If a number reads the same from left to right as from right to left, it is called a 'palindrome'. For example, 3883, 12321 and 25052 are 'palindromes'. Find the smallest four-digit 'palindrome'. (3 marks)
3. 小琪把十個連續正整數乘起來然後除以 100。她所得的餘數是多少？ (3 分)
Kiki multiplied ten consecutive positive integers together and then divided by 100. What will be the remainder? (3 marks)
4. 某平方數的個位數字是 5。求它的十位數字。 (4 分)
The unit digit of a square number is 5. Find its tens digit. (4 marks)
5. 若某正整數 n 除以 8 時的餘數是 3，則 7^n 的個位數字是甚麼？ (4 分)
What is the unit digit of 7^n if the positive integer n leaves a remainder of 3 when divided by 8? (4 marks)
6. 某足球隊進行了三場比賽，結果是一勝一和一負，合共射入了 5 球失了 1 球。那麼在賽和的一場比賽中，該隊射入了多少球？ (4 分)
A soccer team had 3 matches, resulting in 1 win, 1 draw and 1 loss. The team made a total of 5 goals and lost 1 goal. How many goals did the team score in the match which resulted in a draw? (4 marks)
7. 有多少個不同的長方形各邊的長度都是整數且其面積為 2009？（大小為 $a \times b$ 和 $b \times a$ 的長方形視為相同。） (4 分)
How many different rectangles have all side lengths being integers and area 2009? (Rectangles with sizes $a \times b$ and $b \times a$ are regarded to be the same.) (4 marks)

8. P 是 $\triangle ABC$ 內的一點，它和 $\triangle ABC$ 各邊的距離均是 4。若 $\triangle ABC$ 的周界為 60，求 $\triangle ABC$ 的面積。

P is a point inside $\triangle ABC$ such that it is at a distance of 4 from each of the three sides of $\triangle ABC$. If the perimeter of $\triangle ABC$ is 60, find its area.



(5 分)

(5 marks)

9. 設 p, q 為正整數，使得 $pq^2 + p - 6q = 1$ 。求 q 。

(5 分)

Let p, q be positive integers such that $pq^2 + p - 6q = 1$. Find q .

(5 marks)

10. 在首 2009 個正整數中，有多少個的平方根可寫成 $\sqrt{2}$ 的整數倍數？

(5 分)

How many of the first 2009 positive integers have their square roots expressible as an integral multiple of $\sqrt{2}$?

(5 marks)

11. 有多少個正整數小於 10000 且所有數字都不是 0？

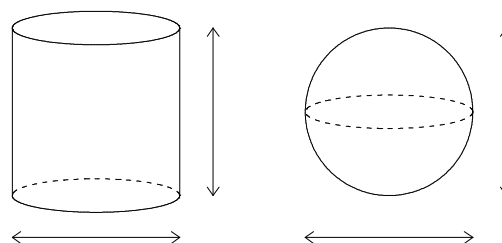
(5 分)

How many positive integers are less than 10000 with all digits non-zero?

(5 marks)

12. 圖中，圓柱體的底直徑、圓柱體的高和球體的直徑全部相同。若圓柱體的體積為 1000，求球體的體積，答案準確至最接近整數。

In the figure, the base diameter of the cylinder, the height of the cylinder as well as the diameter of the sphere are all equal. If the cylinder has volume 1000, find the volume of the sphere correct to the nearest integer.



(5 分)

(5 marks)

13. 設 n 為正整數。若 $\sqrt{n^2 - 21n + 114}$ 是質數，求 n 所有可能值之和。

(6 分)

Let n be a positive integer. If $\sqrt{n^2 - 21n + 114}$ is a prime number, find the sum of all possible values of n .

(6 marks)

14. 設 n 為 2009 位數「2009000...0002009」。求 n^2 的數字之和。 (6 分)

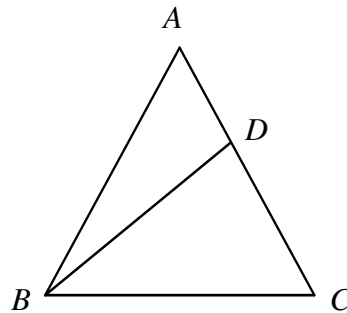
Let n be the 2009-digit number '2009000...0002009'. Find the sum of digits of n^2 . (6 marks)

15. 在一個遊戲中有 100 個盒子，編號為 1 至 100，每個盒子內均有一個紅色、綠色或藍色的球。參加者獲發一張印有 n 個方格的遊戲咭，並可在每個方格填上兩個不同的編號（不同方格中可填上相同的編號）。如果某方格的兩個編號的盒子中的球的顏色相同，參加者便可獲獎。已知有方法填寫遊戲咭以保證獲獎，求 n 的最小可能值。 (6 分)

In a game, there are 100 boxes numbered 1 to 100, each of which contains a red, green or blue ball. A player is given a game card consisting of n boxes, in each of which one can fill in two different numbers (one can fill in the same number in different boxes). If the two numbers in a certain box correspond to two boxes containing two balls of the same colour, the player can win a prize. It is known that there exists a way of filling in the game card to ensure a prize to be won. Find the smallest possible value of n . (6 marks)

16. 圖中， $AB = AC$ ，而 D 是 B 到 AC 的垂足。若 $AD:DC = 5:8$ 而 $BD = 10$ ，求 $\triangle ABC$ 的面積，答案準確至最接近整數。

In the figure, $AB = AC$ and D is the foot of the perpendicular from B to AC . If $AD:DC = 5:8$ and $BD = 10$, find the area of $\triangle ABC$ correct to the nearest integer.



(6 分)

(6 marks)

17. 現有兩個 12 小時制的時鐘。某天正午，兩個鐘都顯示著正確的時間，但第一個鐘每小時走快 12 分鐘，第二個鐘每小時走慢 15 分鐘。小明在之後的每小時都會看看兩個鐘的時間。在多少小時後小明才會首次看到兩個鐘顯示相同的時間？ (6 分)

There are two 12-hour clocks. At noon on a certain day, both clocks are showing the correct time. However, in each subsequent hour the first clock runs 12 minutes fast while the second clock runs 15 minutes slow. Mike checks the two clocks in every subsequent hour. After how many hours will he first see the two clocks showing the same time? (6 marks)

18. 已知 n 是三位正整數，且剛好有三個正因數。若把 n 的數字左右倒轉，所得的三位數也剛好有三個正因數。求 n 所有可能值之和。 (7 分)

Given n is a three-digit positive integer with exactly three positive factors. If the three digits of n are reversed, the new three-digit number formed also has exactly three positive factors. Find the sum of all possible values of n . (7 marks)

19. 某正整數 N 以四進制表示時有 19 個數字。若 N 以八進制表示時，它有多少個數字？ (7 分)

A positive integer N , when expressed in base 4 notation, has 19 digits. What is the number of digits when N is expressed in base 8 notation? (7 marks)

20. 某正多邊形的每隻內角四捨五入至最接近度時為 n° 。問 n 有多少個不同的可能值？ (7 分)

Each interior angle of a regular polygon, when rounded off to the nearest degree, is n° . How many possible values of n are there? (7 marks)

全卷完

END OF PAPER