第八屆培正數學邀請賽

8th Pui Ching Invitational Mathematics Competition

初賽（中二組）

Heat Event (Secondary 2)

時限：1 小時 15 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

1. 本卷共設 20 題，總分爲 100 分。

   There are 20 questions in this paper and the total score is 100.

2. 除特別指明外，本卷內的所有數均爲十進制。

   Unless otherwise stated, all numbers in this paper are in decimal system.

3. 所有答案皆是 0 至 9999 之間的整數（包括 0 和 9999）。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

   All answers are integers between 0 and 9999 (including 0 and 9999). Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

4. 不得使用計算機。

   The use of calculators is not allowed.

5. 本卷的附圖不一定依比例繪成。

   The diagrams in this paper are not necessarily drawn to scale.

1. 求最小的一個四位數，其數字是互不相同的偶數。 （3 分）

Find the smallest four-digit number whose digits are pairwise different even numbers. (3 marks)

2. 設 p 爲質數。已知當 20090124 除以 p 時，餘數是 4。求 p 的最小可能值。 （3 分）

Let p be a prime number. It is known that 20090124 leaves a remainder of 4 when divided by p. Find the smallest possible value of p. (3 marks)

3. 若把點 (3, 4) 繞原點順時針旋轉 2009 個直角，則影像是 (a, b)。求 $a+1000$ 的值。 （3 分）

If the point (3, 4) is rotated about the origin clockwise by 2009 right angles, the image is (a, b). Find the value of $a+1000$. (3 marks)

4. 某長方形的長和闊是不同的整數，且其面積爲 25。求長方形的周界。 （3 分）

A rectangle whose length and width are different integers has area 25. Find its perimeter. (3 marks)

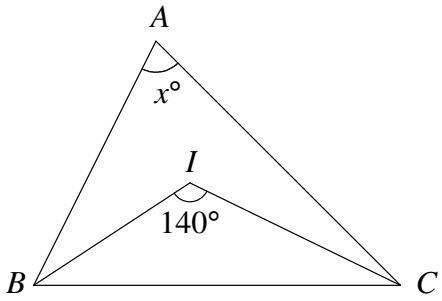
5. 若把分數 $\frac{1}{b}$（其中 b 是正整數）化成小數，則小數點後的第一個數字是 1。問 b 有多少個不同的可能值？ （3 分）

If the fraction $\frac{1}{b}$ (where b is a positive integer) is converted into a decimal, the first digit after the decimal point is 1. How many different possible values of b are there? (3 marks)

6. 小明共有現金 x 元。他把所有金錢以原價購入一批鉛筆，之後以三分之二的價錢把所有鉛筆賣給朋友，收回一筆款項。後來小明得知有人以原價的六分之一出售該款鉛筆，便再把所有金錢向那人買入鉛筆。最後小明以原價把所有鉛筆賣出，結果比原先多了 3600 元。求 x。 （4 分）

Michael has x dollars. He spent the entire amount to buy some pencils at the marked price, and subsequently sold them to his friend at two-thirds of the price to get back some money. Later, he found that someone offered to sell the pencils at one-sixth of the marked price, so he again used all his money to buy the pencils from the seller. He finally sold all the pencils at the marked price and ended up having 3600 dollars more than he originally had. Find x. (4 marks)

7. 圖中，*IB* 和 *IC* 分別是 Δ*ABC* 中的 ∠*B* 和 ∠*C* 的平分線。若 ∠*BIC* = 140° 而 ∠*BAC* = *x*°，求 *x*。 （4 分）

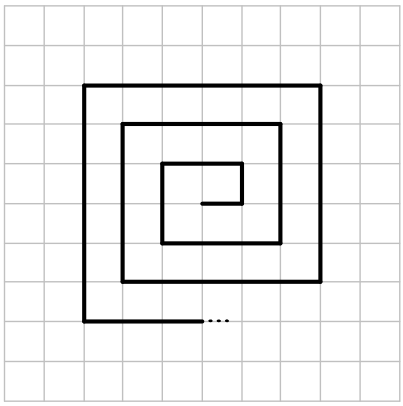


In the figure, *IB* and *IC* are the bisectors of ∠*B* and ∠*C* of Δ*ABC* respectively. If ∠*BIC* = 140° and ∠*BAC* = *x*°, find *x*. (4 marks)

8. 在排球比賽的每局中，對賽雙方開局時均爲 0 分，當其中一隊的得分達 8 分後便會出現第一次技術暫停，而當其中一隊的得分達 16 分後便會出現第二次技術暫停。那麼，在兩次技術暫停之間，兩隊合共最多取得了多少分？ （5 分）

In each game of a volleyball match, both teams start with 0 point. The first technical timeout happens immediately after the score of one of the teams reaches 8 while the second technical timeout happens immediately after the score of one of the teams reaches 16. What is the maximum number of the points obtained by the two teams between the two technical timeouts? (5 marks)

9. 在附圖的方格陣中，相鄰的直線距離均爲 1 單位。若如圖所示從中心位置開始畫一個迷宮形圖案直至其長度達 120 單位，則會畫出多少個直角？ （5 分）



In the grids shown, adjacent lines are 1 unit apart. If a maze-like figure is drawn from the centre as shown to a length of 120 units, how many right angles would have been drawn? (5 marks)

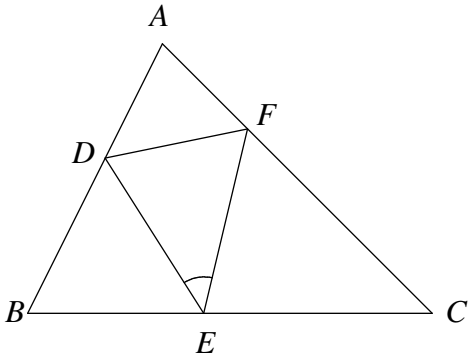
10. 小娜和大娜分別進行同一件工作，兩人的工作效率相同。小娜每工作 15 分鐘便會睡著，要 *n* 分鐘後才睡醒繼續工作，大娜則一直工作沒有睡覺。當小娜完成工作時，剛好又睡著了。若小娜和大娜完成工作所需的時分別是 3 小時和 1.5 小時，求 *n*。 （5 分）

Anna and Diana worked on the same task at the same rate. Anna will fall asleep once she has worked for 15 minutes, and will sleep for *n* minutes before she can wake up and work again. Diana, on the other hand, works continuously without sleeping. When Anna finishes the task, she falls asleep immediately. If Anna and Diana take 3 hours and 1.5 hours to finish the task respectively, find *n*. (5 marks)

11. 小德沿著一個周長 184 米的湖散步。他每走 25 米便會在地上放一塊石子。當他放了 30 塊石子後，最接近的兩塊石子相距多少米（沿湖的周界計算）？ （5 分）

Peter jogs around a lake with perimeter 184 m. He drops a stone on the ground whenever he has walked 25 m. After he has dropped 30 stones, what is the distance (in metres along the perimeter of the lake) between the two closest stones? (5 marks)

12. 圖中，D、E、F 分別是 ΔABC 中 AB、BC 和 CA 上的點，使得 $BD = BE$ 且 $CE = CF$。若 ADF 是等邊三角形而 $\angle DEF = x°$，求 x。 （5 分）



In the figure, D, E, F are points on AB, BC and CA of ΔABC respectively such that $BD = BE$ and $CE = CF$. If ΔADF is equilateral and $\angle DEF = x°$, find x. (5 marks)

13. 在所示的算式中，k 是一個正整數，而兩個空格則分別表示「+」、「−」、「×」、「÷」四個符號的其中之一（兩個符號可以相同）。求 k 所有可能值之和。 （6 分）

$$k \square k \square k = 12$$

In the expression shown, k denotes a positive integer, and each of the two blanks represents one of the symbols '+', '−', '×' and '÷' (the two symbols can be the same). Find the sum of all possible values of k. (6 marks)

14. 當 2009 位數 1000…0001 除以 7 時，餘數是多少？ （6 分）

What is the remainder when the 2009-digit number 1000…0001 is divided by 7? (6 marks)

15. 在首 100 個正整數中，有多少個可被一個兩位質數整除？ （6 分）

How many of the first 100 positive integers are divisible by a two-digit prime number? (6 marks)

16. 設 n 是不大於 2009 的正整數，且 n 和 2009 的最大公因數是 1。問 n 有多少個不同的可能值？ （6 分）

Let n be a positive integer not exceeding 2009. If the H.C.F. of n and 2009 is 1, how many different possible values of n are there? (6 marks)

17. 在一個邊長爲 4 的正方形內有一點 P。把 P 與正方形的四個頂點連起，可把正方形分成四個三角形。若這四個三角形中有剛好三個的面積是質數，則 P 的位置有多少個不同的可能性？ （7 分）

P is a point inside a square of side length 4. When *P* is joined to the four vertices of the square, the square is divided into four triangles. If exactly three of these triangles have an area being a prime number, how many possibilities are there for the position of *P*? (7 marks)

18. 已知 2^{50} 是個 16 位數，其首兩位數字爲 11。那麼 2^{500} 有多少個數字？ （7 分）

Given 2^{50} is a 16-digit number starting with 11. How many digits are there in 2^{500}? (7 marks)

19. 小雄在一條長 n 米的圓形跑道上以均速順時針步行。另有 10 名男孩均勻分佈在跑道上，亦有 10 名女孩均勻分佈在跑道上。每名男孩均以 1 m/s 的速率沿跑道逆時針步行，每名女孩均以 1 m/s 的速率沿跑道順時針步行。若小雄每 40 秒遇上一名男孩迎面而來，每 2 分鐘被一名女孩從後趕上，求 n。 （7 分）

Kenneth walks clockwise along a circular track of length n metres at constant speed. 10 boys are evenly distributed along the track, and 10 girls are evenly distributed along the track. Each boy walks along the track anticlockwise at a speed of 1 m/s, and each girl walks along the track clockwise at a speed of 1 m/s. If Kenneth meets a boy ahead every 40 seconds and a girl from behind every 2 minutes, find n. (7 marks)

20. 六人準備就座拍照，六個座位排成一列。其中兩人是夫婦，他們必須坐在另一人的隔鄰；而另外兩人爲離婚夫婦，他們都不願意坐在另一人的隔鄰。那麼共有多少種方法安排座位？ （7 分）

Six people are to be seated in a row to take a picture. Two of them are couples and they have to sit next to each other, while two others are separated couples and they are not willing to sit next to each other. How many different seating arrangements are possible? (7 marks)

全卷完

END OF PAPER