

第八屆培正數學邀請賽
8th Pui Ching Invitational Mathematics Competition

初賽（中一組）
Heat Event (Secondary 1)

時限：1 小時 15 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

1. 本卷共設 20 題，總分爲 100 分。

There are 20 questions in this paper and the total score is 100.

2. 除特別指明外，本卷內的所有數均爲十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

3. 所有答案皆是 0 至 9999 之間的整數（包括 0 和 9999）。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

All answers are integers between 0 and 9999 (including 0 and 9999). Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

4. 不得使用計算機。

The use of calculators is not allowed.

5. 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

1. 求最大的一個四位數，其所有數字均是質數。 (3 分)
Find the largest four-digit number in which all digits are prime numbers. (3 marks)

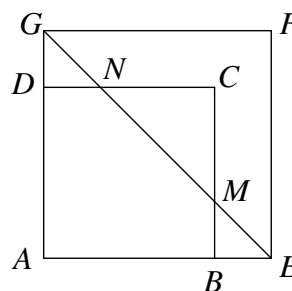
2. 設 n 為正整數。已知當 20090124 除以 n 時，餘數是 4。求 n 的最小可能值。 (3 分)
Let n be a positive integer. It is known that 20090124 leaves a remainder of 4 when divided by n . Find the smallest possible value of n . (3 marks)

3. 若把分數 $\frac{a}{b}$ (其中 a 、 b 都是正整數且 $a < b$) 化成小數，則小數點後的第一個數字是 1。求 b 的最小可能值。 (3 分)
If the fraction $\frac{a}{b}$ (where a, b are positive integers satisfying $a < b$) is converted into a decimal, the first digit after the decimal point is 1. Find the smallest possible value of b . (3 marks)

4. 某長方形各邊的長度都是整數，且其面積為 23。求長方形的周界。 (3 分)
A rectangle with integral side lengths has area 23. Find its perimeter. (3 marks)

5. 圖中， $ABCD$ 和 $AEFG$ 均是正方形，邊長分別是 12 和 16。EG 分別與 BC 和 CD 交於 M 和 N 。求 $\triangle CMN$ 的面積。

In the figure, $ABCD$ and $AEFG$ are squares with side lengths 12 and 16 respectively. EG meets BC at M and CD at N . Find the area of $\triangle CMN$.

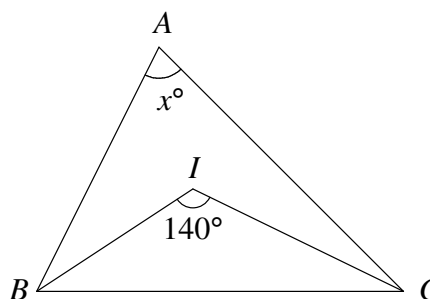


(4 分)

(4 marks)

6. 圖中， IB 和 IC 分別是 $\triangle ABC$ 中的 $\angle B$ 和 $\angle C$ 的平分線。若 $\angle BIC = 140^\circ$ 而 $\angle BAC = x^\circ$ ，求 x 。

In the figure, IB and IC are the bisectors of $\angle B$ and $\angle C$ of $\triangle ABC$ respectively. If $\angle BIC = 140^\circ$ and $\angle BAC = x^\circ$, find x .



(4 分)

(4 marks)

7. 設 n 為大於 1 的整數。若 n^2 和 n^3 的數字之和均為 a 而 n^4 的數字之和是 b ，求 b 的最小可能值。 (4 分)

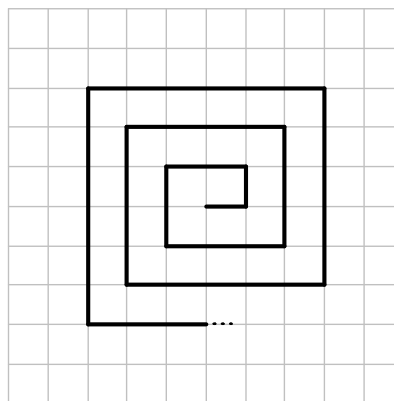
Let n be an integer greater than 1. If the sum of digits of n^2 and n^3 are both a while the sum of digits of n^4 is b , find the smallest possible value of b . (4 marks)

8. 當 2009 位數 $1000\dots0001$ 除以 11 時，餘數是多少？ (5 分)

What is the remainder when the 2009-digit number $1000\dots0001$ is divided by 11? (5 marks)

9. 在附圖的方格陣中，相鄰的直線距離均為 1 單位。若如圖所示從中心位置開始畫一個迷宮形圖案直至其長度達 120 單位，則會畫出多少個直角？

In the grids shown, adjacent lines are 1 unit apart. If a maze-like figure is drawn from the centre as shown to a length of 120 units, how many right angles would have been drawn?

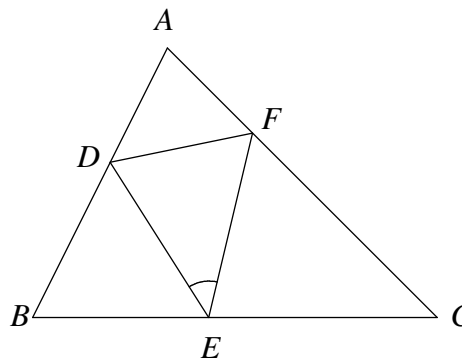


(5 分)

(5 marks)

10. 圖中， D 、 E 、 F 分別是 $\triangle ABC$ 中 AB 、 BC 和 CA 上的點，使得 $BD = BE$ 且 $CE = CF$ 。若 $\triangle ADF$ 是等邊三角形而 $\angle DEF = x^\circ$ ，求 x 。

In the figure, D , E , F are points on AB , BC and CA of $\triangle ABC$ respectively such that $BD = BE$ and $CE = CF$. If $\triangle ADF$ is equilateral and $\angle DEF = x^\circ$, find x .



(5 分)

(5 marks)

11. 一家百貨公司正進行推廣活動，所有貨品照價八折，每買滿 250 元（以折實價計算）更可即時獲扣減 10 元（買滿 500 元可扣減 20 元，如此類推）。小妮打算買 190 支標價每支 10 元的原子筆，那麼她最少要付多少元？ (5 分)

A department store is having a sales promotion in which all goods are sold 20% off the marked price. Furthermore, for every purchase of 250 dollars (at the discounted price), an immediate rebate of 10 dollars will be given (a rebate of 20 dollars is given for a purchase of 500 dollars, and so on). Fanny plans to buy 190 ball pens which are marked at 10 dollars each. What is the minimum amount (in dollars) that she has to pay? (5 marks)

12. 在首 2009 個正整數中，有多少個除了自己以外還有一個四位的正因數？ (5 分)

How many of the first 2009 positive integers have a 4-digit positive factor other than itself? (5 marks)

13. 某個一年一度的數學比賽由初賽和決賽兩部分組成，分別於每年一月至七月（包括首尾兩月）之間兩個不同的星期六舉行，並先舉行初賽。若加入初賽和決賽的日期中的「日」相同這項要求，則在 2009 年，這項比賽的初賽和決賽日期組合有多少個不同的可能性？（已知 2009 年 1 月 1 日是星期四。） (6 分)

An annual mathematics competition consists of a Heat Event and a Final Event, to be held on two different Saturdays between January and July (inclusive) each year, with the Heat Event preceding the Final Event. How many different choices are there for the combination of the two dates for the competition in 2009, if it is further required that the 'days' of the two dates be the same? (It is given that 1st January 2009 is a Thursday.) (6 marks)

14. 在所示的算式中， k 是一個正整數，而兩個空格則分別表示「+」、「-」、「 \times 」、「 \div 」四個符號的其中之一（兩個符號可以相同）。求 k 所有可能值之和。 (6 分)

In the expression shown, k denotes a positive integer, and each of the two blanks represents one of the symbols '+', '-', ' \times ' and ' \div ' (the two symbols can be the same). Find the sum of all possible values of k .

$$k \square k \square k = 12$$

15. 在首 100 個正整數中，有多少個可被一個兩位平方數整除？ (6 分)

How many of the first 100 positive integers are divisible by a two-digit square number? (6 marks)

16. 六人準備就座拍照，六個座位排成一列。其中兩人是夫婦，他們必須坐在另一人的隔鄰；而另外兩人為兄弟，他們亦必須坐在另一人的隔鄰。那麼共有多少種方法安排座位？ (6 分)

Six people are to be seated in a row to take a picture. Two of them are couples and they have to sit next to each other, and so do two others who are brothers. How many different seating arrangements are possible? (6 marks)

17. 已知 2^{50} 是個 16 位數，其首兩位數字為 11。那麼 2^{100} 有多少個數字？ (6 分)
- Given 2^{50} is a 16-digit number starting with 11. How many digits are there in 2^{100} ? (6 marks)

18. 設 $[x]$ 代表不超過 x 的最大整數，例如 $[1.1] = 1$ 、 $[6.9] = 6$ 和 $[5] = 5$ 。已知 $\pi \approx 3.14159265$ ，求 $\left[\pi + \frac{1}{2009}\right] + \left[\pi + \frac{2}{2009}\right] + \cdots + \left[\pi + \frac{2008}{2009}\right]$ 的值。 (7 分)

Let $[x]$ denote the greatest integer not exceeding x . For example, $[1.1] = 1$, $[6.9] = 6$ and $[5] = 5$. Given that $\pi \approx 3.14159265$, find the value of $\left[\pi + \frac{1}{2009}\right] + \left[\pi + \frac{2}{2009}\right] + \cdots + \left[\pi + \frac{2008}{2009}\right]$. (7 marks)

19. 小雄在一條長 n 米的圓形跑道上以均速順時針步行。另有 10 名男孩均勻分佈在跑道上，亦有 10 名女孩均勻分佈在跑道上。每名男孩均以 1 m/s 的速率沿跑道逆時針步行，每名女孩均以 1 m/s 的速率沿跑道順時針步行。若小雄每 40 秒遇上一名男孩迎面而來，每 2 分鐘被一名女孩從後趕上，求 n 。 (7 分)

Kenneth walks clockwise along a circular track of length n metres at constant speed. 10 boys are evenly distributed along the track, and 10 girls are evenly distributed along the track. Each boy walks along the track anticlockwise at a speed of 1 m/s, and each girl walks along the track clockwise at a speed of 1 m/s. If Kenneth meets a boy ahead every 40 seconds and a girl from behind every 2 minutes, find n . (7 marks)

20. 在一個邊長為 4 的正方形內有一點 P 。把 P 與正方形的四個頂點連起，可把正方形分成四個三角形。若這四個三角形中有剛好三個的面積是質數，則 P 的位置有多少個不同的可能性？ (7 分)

P is a point inside a square of side length 4. When P is joined to the four vertices of the square, the square is divided into four triangles. If exactly three of these triangles have an area being a prime number, how many possibilities are there for the position of P ? (7 marks)

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END OF PAPER