第八屆培正數學邀請賽

8th Pui Ching Invitational Mathematics Competition

決賽（中三組）

Final Event (Secondary 3)

時限：2 小時

Time allowed: 2 hours

參賽者須知：

Instructions to Contestants:

1. 本卷共設 20 題，總分為 100 分。

   There are 20 questions in this paper and the total score is 100.

2. 除特別指明外，本卷內的所有數均為十進制。

   Unless otherwise stated, all numbers in this paper are in decimal system.

3. 除特別指明外，所有答案須以數字的真確值表達，並化至最簡。不接受近似值。

   Unless otherwise stated, all answers should be given in exact numerals in their simplest form. No approximation is accepted.

4. 把所有答案填在答題紙指定的空位上。毋須呈交計算步驟。

   Put your answers on the space provided on the answer sheet. You are not required to hand in your steps of working.

5. 不得使用計算機。

   The use of calculators is not allowed.

6. 本卷的附圖不一定依比例繪成。

   The diagrams in this paper are not necessarily drawn to scale.

第 1 至第 4 題，每題 3 分。

Questions 1 to 4 each carries 3 marks.

1. 網站「數學資料庫」於 2003 年 3 月 14 日成立。2009 年 3 月 14 日（星期六）是「數學資料庫」成立六週年的紀念日。「數學資料庫」下一次在星期六的週年紀念日出現在哪一年？

   The website 'Mathematical Database' was founded on 14th March 2003. On Saturday 14th March 2009, 'Mathematical Database' celebrates its 6th anniversary. In which year will 'Mathematical Database' next celebrate its anniversary on a Saturday?

2. 求 2009^{2009} 的最後兩位數字。

   Find the last two digits of 2009^{2009}.

3. 設 m 是三位正整數，m 的數字之和是 n 而 n 的數字之和是 8。求 m 的最大可能值。

   Let m be a three-digit positive integer with sum of digits n. If the sum of digits of n is 8, find the greatest possible value of m.

4. 某村莊有 600 人，其中 x 人永遠說真話，其餘的人永遠說謊話。某天，那 600 人圍成一圈，然後齊聲說：「在向我右邊數的首兩個人當中，最少一個是說謊話的人」。求 x。

   There are 600 people in a village. Among them x people always tell the truth and the others always lie. One day, the 600 people formed a circle and said together, 'of the two people immediately on my right, at least one is a liar'. Find x.

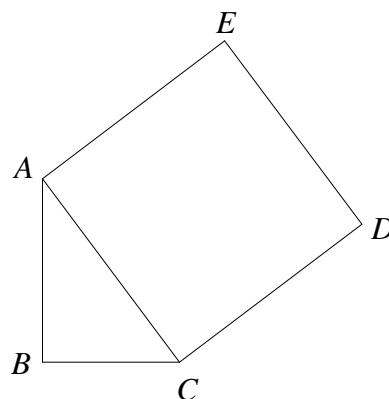
第 5 至第 8 題，每題 4 分。

Questions 5 to 8 each carries 4 marks.

5. 圖中，D、E、F 分別是 BC、CA 和 AB 的中點，P、Q 分別是 ΔBDF 和 ΔCDE 的重心。若 $BC = 60$，求 PQ。

   In the figure, D, E, F are the mid-points of BC, CA and AB respectively, and P, Q are the centroids of ΔBDF and ΔCDE respectively. If $BC = 60$, find PQ.
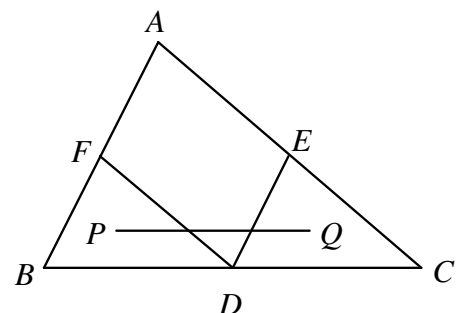
6. 圖中，*ABC* 是直角三角形，*B* 是直角，而 *ACDE* 是正方形。若 *BC* = 3 而 *AC* = 5，求 *BE*。

   In the figure, Δ*ABC* is right-angled at *B* and *ACDE* is a square. If *BC* = 3 and *AC* = 5, find *BE*.


7. 下表詳列了某城市的用水收費。該市的一家蔬菜店某月的用水量為 *n* 立方米，而該月平均每立方米的水費為 10 元。求 *n* 的值。

   The following table lists the water charges of a city. The water usage of a vegetable shop in the city is *n* cubic metres in a certain month and the average charge of its water usage that month is 10 dollars per cubic metre. Find the value of *n*.

每月用水收費表	Table for monthly fee of water supply
首 10 立方米 ....免費	The first 10 cubic metres.................................... Free
次 20 立方米 ....每立方米 2 元	The next 20 cubic metres ...2 dollars per cubic metre
次 30 立方米 ....每立方米 5 元	The next 30 cubic metres ...5 dollars per cubic metre
次 40 立方米 ....每立方米 9 元	The next 40 cubic metres ...9 dollars per cubic metre
...................以後每立方米 15 元	.... 15 dollars per cubic metre thereafter

8. 圖中，*M* 是 *BC* 的中點，*N* 是 *M* 到 *AC* 的垂足。若 *AB* = *AC* = 50 而 *BC* = 60，求 Δ*MNC* 的面積。

   In the figure, *M* is the mid-point of *BC* and *N* is the foot of the perpendicular from *M* to *AC*. If *AB* = *AC* = 50 and *BC* = 60, find the area of Δ*MNC*.
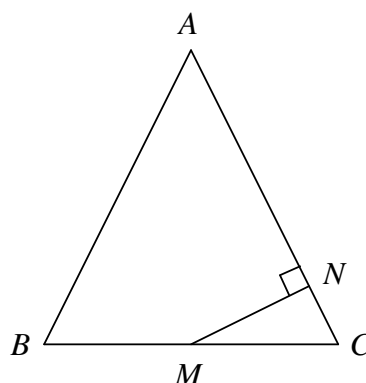
9. 現容許在整數 11111111 的兩個連續「1」字之間加上「+」號來組成不同的正整數，例如：我們可以得到 $1111+1111=2222$ 和 $1+11+1+1+11+1=26$。可以這樣得出來的正整數稱爲「好數」。求所有三位「好數」之和。

   From the number 11111111, one is allowed to insert the symbol '+' between two consecutive 1's to form various positive integers. For instance, one may get $1111+1111=2222$ and $1+11+1+1+11+1=26$. Positive integers which can be formed in this way are said to be 'good'. Find the sum of all 'good' three-digit numbers.

10. 某星球是半徑爲 $\dfrac{10}{\pi}$ 公里的球體，它使用與地球相同的方向表示法。俊熙在該星球的赤道上的一點出發，沿赤道繞星球轉四分之一個圈後往北走 1 公里，最後在星球表面上沿最短的路徑返回起點。他全程共走了多少公里？

    A planet is a sphere of radius $\dfrac{10}{\pi}$ km. It adopts the same representation of directions as on the Earth. Bob started his travel from a point on the equator of the planet. He orbited a quarter of the planet along the equator, and then went north for 1 km. Finally, he returned to his starting point via the shortest path on the surface of the planet. What is the total distance he travelled (in km)?

11. 圖中顯示一條除式，但當中有些數字留空了。求該六位的被除數。

    The figure shows a division, but some digits are left out. Find the six-digit dividend.

12. 已知 $\left(n+\dfrac{x}{2n}\right)^2 = n^2 + x + \dfrac{x^2}{4n^2}$。求最接近 $\sqrt{64340000}$ 的整數。

    Given $\left(n+\dfrac{x}{2n}\right)^2 = n^2 + x + \dfrac{x^2}{4n^2}$. Find the integer closest to $\sqrt{64340000}$.

第 13 至第 16 題，每題 6 分。

Questions 13 to 16 each carries 6 marks.

13. 求 0 至 1 之間所有分母是 2009 的最簡分數之和。

    Find the sum of all irreducible fractions between 0 and 1 whose denominator is equal to 2009.

14. 小琪到 X 國旅行 7 天。該國有四個城市，A、B、C、D。每天，小琪都會遊覽其中一個城市，而且不會連續兩天遊覽同一城市。已知小琪在第 1 天和第 7 天都是遊覽城市 A，問小琪這 7 天的行程有多少個不同的可能性？

    Vicky goes for a 7-day trip in Country X. There are four cities, A, B, C and D, in the country. Each day Vicky visits one city, and she will not visit the same city on two consecutive days. Given that Vicky visits City A on both Day 1 and Day 7, how many different possible itineraries are there for her 7-day trip?

15. 如果某數由左至右和由右至左看皆相同，我們稱這個數是「回文數」。例如 3883、12321 和 25052 都是「回文數」。求 2600 和 3000 之間的一個整數 n，使得 n^2 是「回文數」。

    If a number reads the same from left to right as from right to left, it is called a 'palindrome'. For example, 3883, 12321 and 25052 are 'palindromes'. Find an integer n between 2600 and 3000 for which n^2 is a 'palindrome'.

16. 有多少個十位正整數的每個數字皆是 1 或 2，且有最少五個連續數字相同？

    How many ten-digit positive integers are there such that each digit is either 1 or 2, and that at least five consecutive digits are the same?

第 17 至第 20 題，每題 7 分。

Questions 17 to 20 each carries 7 marks.

17. 求 $1^3 + 11^3 + 21^3 + \cdots + 20091^3 + 20101^3$ 除以 2009 時的餘數。

    Find the remainder when $1^3 + 11^3 + 21^3 + \cdots + 20091^3 + 20101^3$ is divided by 2009.

18. 已知 a、b、c、d、e、f、g 是七個互不相同的正整數，其中 $a<b$、$a<c$ 而 $d<e<f$。小敏希望把七個數從小至大排列，她每次會選其中兩個數，然後小婷會告訴小敏兩個數中哪個較大。這個過程最少要進行多少次，小敏才可保證正確地排序？
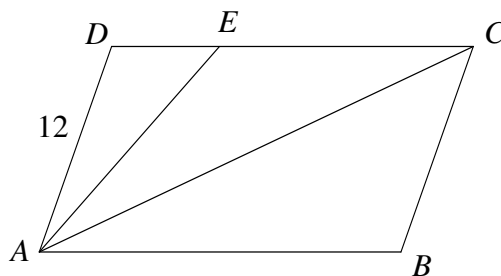
Let a, b, c, d, e, f, g be seven pairwise different positive integers such that $a<b$, $a<c$ and $d<e<f$. Mandy wishes to arrange these numbers in ascending order. Each time she picks two of the numbers and Tiffany will tell her which of the two numbers is larger. At least how many times must this process be carried out in order that Mandy can ensure a correct arrangement?

19. 設 N 是正整數。若 $\dfrac{N}{2}$ 是平方數、$\dfrac{N}{3}$ 是立方數、$\dfrac{N}{5}$ 是某整數的五次方，且 N 除以 11 時的餘數是 k，求 k 所有可能值之和。

Let N be a positive integer. If $\dfrac{N}{2}$ is a square number, $\dfrac{N}{3}$ is a cubic number and $\dfrac{N}{5}$ is the fifth power of an integer, and that N leaves a remainder of k when divided by 11, find the sum of all possible values of k.

20. 圖中，$ABCD$ 是平行四邊形，其中 $\angle CAD = 2\angle CAB$。$\angle CAD$ 的角平分線交 CD 於 E。若 $AD = 12$ 而 $CE = DE + 2$，求 AC。

In the figure, $ABCD$ is a parallelogram with $\angle CAD = 2\angle CAB$. The bisector of $\angle CAD$ meets CD at E. If $AD = 12$ and $CE = DE + 2$, find AC.

全卷完

END OF PAPER