第八屆培正數學邀請賽

8th Pui Ching Invitational Mathematics Competition

決賽（中二組）

Final Event (Secondary 2)

時限：2 小時

Time allowed: 2 hours

參賽者須知：

Instructions to Contestants:

1. 本卷共設 20 題，總分爲 100 分。

   There are 20 questions in this paper and the total score is 100.

2. 除特別指明外，本卷內的所有數均爲十進制。

   Unless otherwise stated, all numbers in this paper are in decimal system.

3. 除特別指明外，所有答案須以數字的真確値表達，並化至最簡。不接受近似值。

   Unless otherwise stated, all answers should be given in exact numerals in their simplest form. No approximation is accepted.

4. 把所有答案塡在答題紙指定的空位上。毋須呈交計算步驟。

   Put your answers on the space provided on the answer sheet. You are not required to hand in your steps of working.

5. 不得使用計算機。

   The use of calculators is not allowed.

6. 本卷的附圖不一定依比例繪成。

   The diagrams in this paper are not necessarily drawn to scale.

第 1 至第 4 題，每題 3 分。

Questions 1 to 4 each carries 3 marks.

1. 網站「數學資料庫」於 2003 年 3 月 14 日成立。2009 年 3 月 14 日（星期六）是「數學資料庫」成立六週年的紀念日。「數學資料庫」下一次在星期六的週年紀念日出現在哪一年？

   The website ‘Mathematical Database’ was founded on 14th March 2003. On Saturday 14th March 2009, ‘Mathematical Database’ celebrates its 6th anniversary. In which year will ‘Mathematical Database’ next celebrate its anniversary on a Saturday?

2. 三個圓的半徑分別是 a、b、c，而它們的周界分別是 2007、2009 和 x。若 $2b = a + c$，求 x。

   The radii of three circles are a, b and c respectively, and their perimeters are 2007, 2009 and x respectively. If $2b = a + c$, find x.

3. 求 1111^3 除以 2222 時的餘數。

   Find the remainder when 1111^3 is divided by 2222.

4. 現有一個正六邊形，並容許從中選三個頂點來組成三角形。那麼共可組成多少個不同的三角形？（兩個全等的三角形視為同一個三角形。）

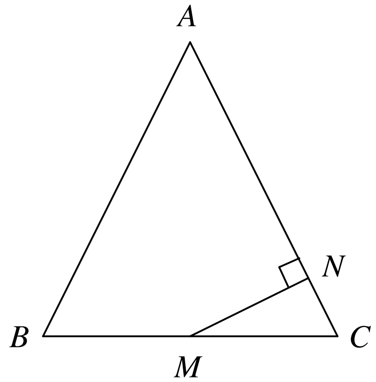
   If one is allowed to take three vertices from a given regular hexagon to form a triangle, how many different triangles may be formed? (Two triangles are regarded to be the same if they are congruent.)

第 5 至第 8 題，每題 4 分。

Questions 5 to 8 each carries 4 marks.

5. 圖中，M 是 BC 的中點，N 是 M 到 AC 的垂足。若 $AB = AC = 50$ 而 $BC = 60$，求 ΔMNC 的面積。

In the figure, M is the mid-point of BC and N is the foot of the perpendicular from M to AC. If $AB = AC = 50$ and $BC = 60$, find the area of ΔMNC.



6. 下表詳列了某城市的用水收費。該市某商業大樓共擁有 10 家商店，它們的水費都獨立計算。已知各商店某月的用水量之和爲 p 立方米，而總水費爲 2008 元，求 p 的最大可能值。

The following table lists the water charges of a city. There are 10 shops in a commercial building in the city. Each shop pays for their water usage individually. It is known that in a certain month the sum of the water usages of the shops is p cubic metres and the total charges for water supply is 2008 dollars. Find the greatest possible value of p.

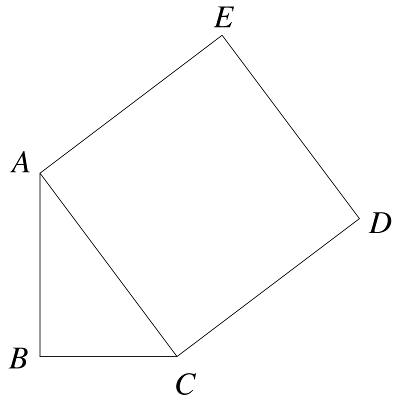
每月用水收費表	Table for monthly fee of water supply
首 10 立方米 ....免費	The first 10 cubic metres.................................. Free
次 20 立方米 .... 每立方米 2 元	The next 20 cubic metres ...2 dollars per cubic metre
次 30 立方米 .... 每立方米 5 元	The next 30 cubic metres ...5 dollars per cubic metre
次 40 立方米 .... 每立方米 9 元	The next 40 cubic metres ...9 dollars per cubic metre
.... 以後每立方米 15 元	.... 15 dollars per cubic metre thereafter

7. 子峰在黑板上以十進制寫下 2^{2009}。之後他算出黑板上的數的數字之和再乘以 3，然後把黑板上的數擦掉，寫上他的答案。假如子峰把這個過程重覆 2009 次，求黑板上最後出現的整數。

Alan wrote the number 2^{2009} on the board in decimal notation. He then computed the sum of digits of the number on the board and then multiplied by 3. After that he erased the number on the board and replaced it by his answer. If this process is repeated 2009 times, what would be the number that eventually appeared on the board?

8. 圖中，ABC 是直角三角形，B 是直角，而 $ACDE$ 是正方形。若 $BC = 3$ 而 $AC = 5$，求 BE。

In the figure, ΔABC is right-angled at B and $ACDE$ is a square. If $BC = 3$ and $AC = 5$, find BE.

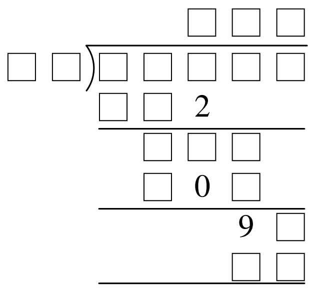


第 9 至第 12 題，每題 5 分。

Questions 9 to 12 each carries 5 marks.

9. 圖中顯示一條除式，但當中有些數字留空了。求該五位的被除數。

The figure shows a division, but some digits are left out. Find the five-digit dividend.

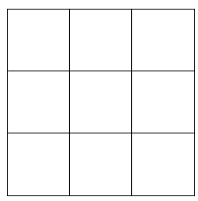


10. 已知 $\left(n+\frac{x}{2n}\right)^2 = n^2 + x + \frac{x^2}{4n^2}$。求最接近 $\sqrt{64340000}$ 的整數。

Given $\left(n+\frac{x}{2n}\right)^2 = n^2 + x + \frac{x^2}{4n^2}$. Find the integer closest to $\sqrt{64340000}$.

11. 丞德在圖中的 3×3 方格表裏每小格填進一個正整數（容許重覆）。填好後，他算算每直行和橫行的和，發現六個和都是互不相同的合成數。若方格表裏的九個數之和是 n，求 n 的最小可能值。

Ted filled every cell of a 3×3 grid in the figure with a positive integer (repetition is allowed). After that, he computed the sum in each row and column and found that these six sums are pairwise distinct composite numbers. If the sum of the nine numbers in the grid is n, find the smallest possible value of n.



12. 如果某數由左至右和由右至左看皆相同，我們稱這個數是「回文數」。例如 3883、12321 和 25052 都是「回文數」。求 2250 和 2500 之間的一個整數 n，使得 n^2 是「回文數」。

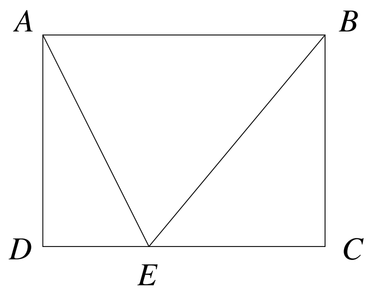
If a number reads the same from left to right and from right to left, it is called a ‘palindrome’. For example, 3883, 12321 and 25052 are ‘palindromes’. Find an integer n between 2250 and 2500 for which n^2 is a ‘palindrome’.

第 13 至第 16 題，每題 6 分。

Questions 13 to 16 each carries 6 marks.

13. 圖中，$ABCD$ 是長方形，E 是 CD 上的一點，而 BC 的長度爲 x。已知當 E 在 CD 上（包括端點）移動時，$AE+BE$ 的值最小是 14，最大是 16。求 x 所有可能值之和。

In the figure, $ABCD$ is a rectangle, E is a point on CD and the length of BC is x. It is known that as E moves along CD (including the end-points), the minimum value of $AE+BE$ is 14 and the maximum value is 16. Find the sum of all possible values of x.



14. 有多少個十位正整數的每個數字皆是 1 或 2，且有最少五個連續數字相同？

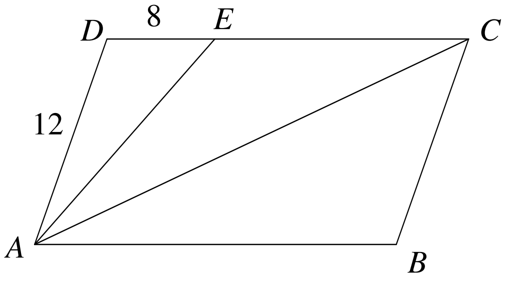
How many ten-digit positive integers are there such that each digit is either 1 or 2, and that at least five consecutive digits are the same?

15. 設 $[x]$ 代表不超過 x 的最大整數，例如 $[1.1] = 1$、$[6.9] = 6$ 和 $[5] = 5$。小儀選了一個正數 M，並發現只要 a、b 是正數且 $[a] \neq [b]$ 時，則 $\frac{a^2-b^2}{[a]^2-[b]^2}$ 的值總是小於 M。求 M 的最小可能值。

Let $[x]$ denote the greatest integer not exceeding x. For example, $[1.1] = 1$, $[6.9] = 6$ and $[5] = 5$. Sharon has chosen a positive number M and she finds that the value of $\frac{a^2-b^2}{[a]^2-[b]^2}$ is always less than M whenever a, b are positive numbers satisfying $[a] \neq [b]$. Find the smallest possible value of M.

16. 圖中，$ABCD$ 是平行四邊形，其中 $\angle CAD = 2\angle CAB$。$\angle CAD$ 的角平分線交 CD 於 E。若 $AD = 12$ 而 $DE = 8$，求 AC。

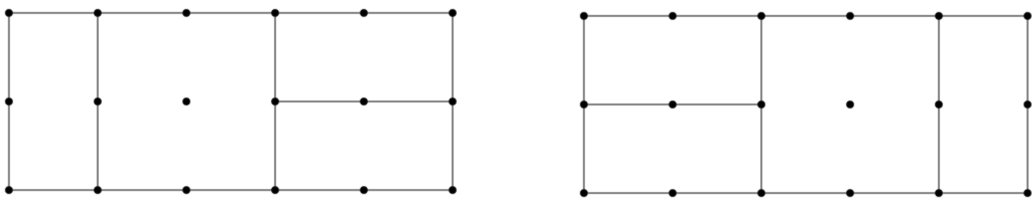
In the figure, $ABCD$ is a parallelogram with $\angle CAD = 2\angle CAB$. The bisector of $\angle CAD$ meets CD at E. If $AD = 12$ and $DE = 8$, find AC.



第 17 至第 20 題，每題 7 分。

Questions 17 to 20 each carries 7 marks.

17. 俊華把一些 1×2、2×1 和 2×2 大小的咭片砌成 $2\times n$ 大小的長方形（當中咭片不許重疊）。下圖顯示了兩個 2×5 大小的長方形，其中把左方的長方形繞其中心旋轉 180° 便可得到右方的長方形。若一個 $2\times n$ 大小的長方形繞其中心旋轉 180° 所得的影像與原圖形相同，那麼我們說它是「對稱」的。俊華有多少種方法砌出一個 2×12 的「對稱」長方形？



John uses some 1×2, 2×1 and 2×2 cardboards to make up (without overlapping) $2\times n$ rectangles. Two 2×5 rectangles are shown above, with the one on the right obtained from the one on the left by a rotation of 180° about the centre of the figure. A $2\times n$ rectangle is said to be ‘symmetric’ if a rotation of 180° about its centre gives an image which is identical to the original rectangle. In how many different ways can John make a ‘symmetric’ 2×12 rectangle?

18. 設 N 是正整數。若 $\frac{N}{2}$ 是平方數、$\frac{N}{3}$ 是立方數、$\frac{N}{5}$ 是某整數的五次方，且 N 除以 11 時的餘數是 k，求 k 所有可能值之和。

Let N be a positive integer. If $\frac{N}{2}$ is a square number, $\frac{N}{3}$ is a cubic number and $\frac{N}{5}$ is the fifth power of an integer, and that N leaves a remainder of k when divided by 11, find the sum of all possible values of k.

19. 如果一個正整數中任何兩個連續數字均組成一個兩位質數，則該正整數稱爲「好數」。例如：由於 17、73、37 和 73 均是兩位質數，故 17373 是「好數」。有多少個六位正整數是「好數」？

If any two consecutive digits of a positive integer form a two-digit prime number, we say that the positive integer is 'good'. For instance, 17373 is 'good' since 17, 73, 37 and 73 are all two-digit prime numbers. How many six-digit positive integers are 'good'?

20. 設 m、n 爲正整數。當 $\frac{m}{n}$ 寫成小數時，小數點後其中三個連續的數字依次是 1、6、7。求 n 的最小可能值。

Let m, n be positive integers. When $\frac{m}{n}$ is expressed as a decimal, three consecutive digits after the decimal point are 1, 6, 7 in order. Find the smallest possible value of n.

全卷完

END OF PAPER