第七屆培正數學邀請賽

7th Pui Ching Invitational Mathematics Competition

初賽（中四組）

Heat Event (Secondary 4)

時限：1 小時 15 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

1. 本卷共設 20 題，總分爲 100 分。

   There are 20 questions in this paper and the total score is 100.

2. 除特別指明外，本卷內的所有數均爲十進制。

   Unless otherwise stated, all numbers in this paper are in decimal system.

3. 所有答案皆是 0 至 9999 之間的整數（包括 0 和 9999）。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

   All answers are integers between 0 and 9999 (including 0 and 9999). Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

4. 不得使用計算機。

   The use of calculators is not allowed.

5. 本卷的附圖不一定依比例繪成。

   The diagrams in this paper are not necessarily drawn to scale.

1. 已知 $126^2 = 15876$，則 20080126^2 的最後四位數字是甚麼？ （2分）

Given that $126^2 = 15876$, what are the last four digits of 20080126^2? (2 marks)

2. 已知 n 是合成數，它既不可被 2 整除，亦不可被 3 整除。求 n 的最小可能值。 （3分）

Given that n is a composite number which is divisible by neither 2 nor 3. Find the smallest possible value of n. (3 marks)

3. 平面上有 2008 條直線，它們的斜率都是整數且互不相同。問所有直線的交角中，最多有幾隻直角？ （3分）

On the plane there are 2008 straight lines. They have pairwise distinct integral slopes. What is the maximum number of right angles formed from the lines? (3 marks)

4. 若 a 和 b 爲滿足 $|502a-1000| = -\sqrt{b-2008}$ 的實數，求 ab。 （3分）

If a and b are real numbers satisfying $|502a-1000| = -\sqrt{b-2008}$, find ab. (3 marks)

5. 某數學競賽規定得獎人數不超過參賽人數的四分之一，且最少一人得獎。若比賽有 9 人參加，則得獎名單有多少個不同的可能組合？ （4分）

In a mathematical competition, it is specified that the number of awardees must not exceed one-fourth of the number of contestants, and that there should be at least one awardee. If 9 people join the competition, how many different possible combinations of the list of awardees are there? (4 marks)

6. 當 $\underbrace{123123123\ldots123}_{\text{2008 個「123」}}$ 除以 7 時，餘數是多少？ （4分）

What is the remainder when $\underbrace{123123123\ldots123}_{\text{2008 copies of '123'}}$ is divided by 7? (4 marks)

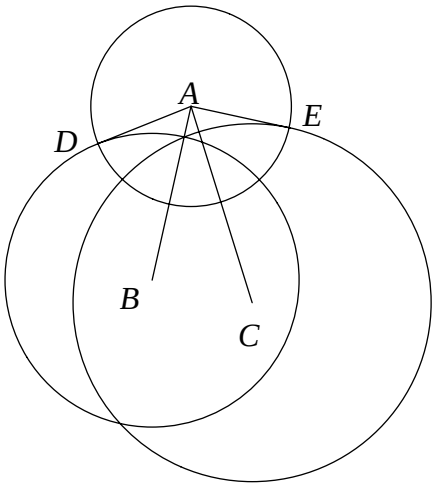
7. 某個十二邊形是 k 重旋轉對稱圖形，其中 $1 < k < 12$。求 k 所有可能值之和。 （4分）

A 12-sided polygon possesses rotational symmetry of order k, where $1 < k < 12$. Find the sum of all possible values of k. (4 marks)

8. 方程 $x^6 - 2008x^4 - 126x^2 = 0$ 有多少個不同的實根？ （4分）

How many different real roots are there to the equation $x^6 - 2008x^4 - 126x^2 = 0$? (4 marks)

9. 圖中，A、B、C 分別爲三個圓的圓心，而三個圓的半徑則依次爲 3、4 和 5。AD 和 AE 都是第一個圓的半徑，其中 AD 與第二個圓相切於 D，而 AE 則與第三個圓相切於 E。求 $AC^2 - AB^2$。 （5分）



In the figure, A, B, C are the centres of the three circles, and the radii of the circles are 3, 4 and 5 respectively. Both AD and AE are radii of the first circle, where AD is tangent to the second circle at D while AE is tangent to the third circle at E. Find $AC^2 - AB^2$. (5 marks)

10. 若某月的第 13 天是星期五，則那天稱爲「黑色星期五」。小婷在某個黑色星期五出生，之後到她 n 個月大時才再出現黑色星期五。求 n 的最大可能值。 （5分）

If the 13th day of a month is Friday, that day is called 'black Friday'. Elaine was born on a black Friday, and it was not until she was n months old when another black Friday appeared. Find the greatest possible value of n. (5 marks)

11. 一個非長方形的四邊形的四隻內角分別爲 $p°$、$q°$、$r°$ 和 $s°$，其中 p、q、r、s 爲正整數，且它們的最大公因數爲 d。求 d 的最大可能值。 （5分）

A non-rectangular quadrilateral has interior angles $p°$, $q°$, $r°$ and $s°$, where p, q, r, s are positive integers with H.C.F. d. Find the greatest possible value of d. (5 marks)

12. 設 $N = 13 \times 17 \times 41 \times 829 \times 56659712633$。已知 N 是一個 18 位數，而 0 至 9 十個數字當中其中九個在 N 的 18 個數字裏各出現了兩次。求 N 的數字之和。 （5分）

Let $N = 13 \times 17 \times 41 \times 829 \times 56659712633$. It is known that N is an 18-digit number, with nine of the ten digits from 0 to 9 each appearing twice. Find the sum of the digits of N. (5 marks)

13. 某袋子中有 m 個紅球和 n 個藍球，其中 $m+n=100$。每個球上都有一個不超過 100 的正整數，沒有兩個球上的整數相同，且所有紅球上的整數之和等於所有藍球上的整數之和。求 mn 的最小可能值。（6分）

In a bag there are m red balls and n blue balls, where $m+n=100$. On each ball there is a positive integer not exceeding 100, the numbers on the balls are pairwise distinct and the sum of the numbers on all red balls is equal to the sum of the numbers on all blue balls. Find the smallest possible value of mn. (6 marks)

14. 小盈不斷投擲一枚硬幣，直至「連續三次擲得正面」或「擲得反面後立即擲得正面」便停止。若因爲「擲得反面後立即擲得正面」而停止的概率是 $\frac{n}{1000}$，求 n。（6分）

Kendra keeps tossing a coin until she obtains 'three consecutive heads' or 'a head immediately following a tail'. If the probability that she stops by getting 'a head immediately following a tail' is $\frac{n}{1000}$, find n. (6 marks)

15. 在所示的加法中，每個字母代表一個 0 至 9 的不同數字。求 FOUR 所代表的四位數的最大可能值。（6分）

In the addition shown, each letter represents a different integer from 0 to 9. Find the greatest possible value of the four-digit number represented by FOUR. (6 marks)

$$\begin{array}{rcccc} & F & O & U & R \\ + & F & O & U & R \\ \hline E & I & G & H & T \end{array}$$

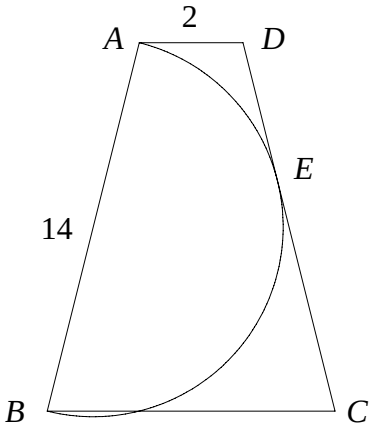
16. 一個直立的圓柱形量筒盛了高 n cm 的水，其中 $n>12$。已知如果在量筒上開洞，只要水面仍在洞之上時，水便會自該洞以均速流走，並且每個洞的水流速度都相等。當在筒底、高 5 cm 和高 12 cm 處各開一個洞，則水會在 5 分鐘後全部流走。當在筒底、高 5 cm、高 9 cm 和高 12 cm 處各開一個洞，則水會在 4 分鐘全部流走。求 n 的值。（7分）

An upright cylindrical measuring cylinder is filled with water to n cm high, where $n>12$. It is known that if holes are drilled on the cylinder, water will flow away from the hole at a constant rate as long as the water level is above the hole. The flow rates in all holes are equal. When one hole is drilled each at the base, 5 cm high and 12 cm high, all water will flow away in 5 minutes. When one hole is drilled each at the base, 5 cm high, 9 cm high and 12 cm high, all water will flow away in 4 minutes. Find the value of n. (7 marks)

17. 某村莊有四名警察。他們分別每 3 天、4 天、11 天和 37 天到村莊巡邏一次。最多有連續多少天村莊都有警察巡邏？ （7 分）

There are four policemen in a village. They patrol in the village once every 3, 4, 11 and 37 days respectively. What is the maximum number of consecutive days in which there is policeman patrolling in the village? (7 marks)

18. 圖中，*ABCD* 是個等腰梯形，其中 $AD = 2$、$AB = DC = 14$，且以 *AB* 爲直徑的半圓與 *CD* 相切於 *E*。求 *ABCD* 的面積。 （7 分）

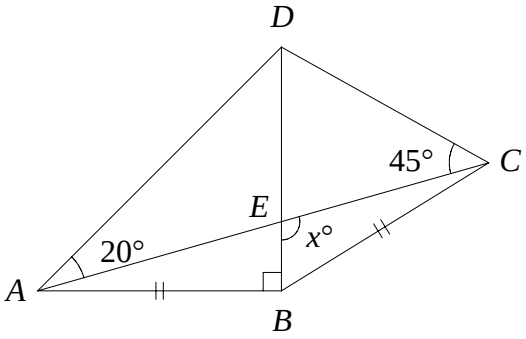


In the figure, *ABCD* is an isosceles trapezium with $AD = 2$ and $AB = DC = 14$. Furthermore, a semi-circle with *AB* as diameter is tangent to *DC* at *E*. Find the area of *ABCD*. (7 marks)

19. 設 Γ 爲一個以 *O* 爲圓心且半徑爲 2008 的圓，*P* 則是滿足 $OP = 2007$ 的一點。問 Γ 有多少條穿過 *P* 且長度爲整數的弦（包括過 *P* 的直徑）？ （7 分）

Let Γ be a circle with radius 2008 centred at *O*, and *P* be a point such that $OP = 2007$. How many chords of Γ passing through *P* (including the diameter through *P*) have integral length? (7 marks)

20. 如圖所示，四邊形 *ABCD* 的對角線 *AC* 和 *BD* 相交於 *E*。若 $AB = BC$、$\angle DAC = 20°$、$\angle DCA = 45°$、$\angle DBA = 90°$ 且 $\angle BEC = x°$，求 *x*。 （7 分）



In the figure, the diagonals *AC* and *BD* of quadrilateral *ABCD* meet at *E*. If $AB = BC$, $\angle DAC = 20°$, $\angle DCA = 45°$, $\angle DBA = 90°$ and $\angle BEC = x°$, find *x*. (7 marks)

全卷完

END OF PAPER