第七屆培正數學邀請賽

7th Pui Ching Invitational Mathematics Competition

初賽（中三組）

Heat Event (Secondary 3)

時限：1 小時 15 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

1. 本卷共設 20 題，總分爲 100 分。

   There are 20 questions in this paper and the total score is 100.

2. 除特別指明外，本卷內的所有數均爲十進制。

   Unless otherwise stated, all numbers in this paper are in decimal system.

3. 所有答案皆是 0 至 9999 之間的整數（包括 0 和 9999）。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

   All answers are integers between 0 and 9999 (including 0 and 9999). Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

4. 不得使用計算機。

   The use of calculators is not allowed.

5. 本卷的附圖不一定依比例繪成。

   The diagrams in this paper are not necessarily drawn to scale.

1. 某 16 個連續正整數之和爲 2008。求當中最小的整數。 （3 分）

The sum of 16 consecutive positive integers is 2008. Find the smallest integer among them. (3 marks)

2. 某圓形的周界爲 $k\pi$ cm、面積爲 $k\pi$ cm^2。求 k。 （3 分）

A circle has perimeter $k\pi$ cm and area $k\pi$ cm^2. Find k. (3 marks)

3. 首 20080126 個正整數中共有 k 個平方數。求 k，答案準確至最接近百位。 （3 分）

There are k square numbers among the first 20080126 positive integers. Find k correct to the nearest hundred. (3 marks)

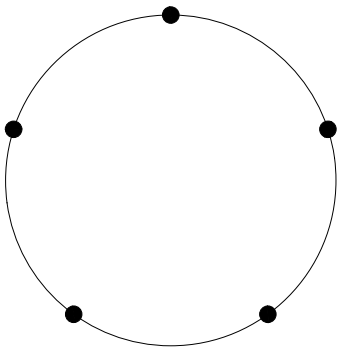
4. 在小於 100 的質數中，有多少個的數字之和是偶數？ （3 分）

Amongst all prime numbers less than 100, how many have an even sum of digits? (3 marks)

5. 某數學競賽的試卷共設 20 題，滿分爲 100 分，而每題的佔分都是 2 至 10 之間的整數（包括 2 和 10）。該試卷最多可以有多少題佔 7 分的題目？ （4 分）

There are 20 questions in the paper of a mathematical competition. The full mark is 100 and the score carried by each question is an integer between 2 and 10 (inclusive). What is the maximum number of questions that may carry 7 marks? (4 marks)

6. 圖中，圓周上的五點把圓周分成五等份。若要從五點中選三點組成等腰三角形，共有多少種選法？ （4 分）



In the figure, the five points on the circumference divides the circumference into five equal parts. If three of the five points are to be chosen to form an isosceles triangle, how many choices are there? (4 marks)

7. 某個十二邊形是 k 重旋轉對稱圖形，其中 $1<k<12$。求 k 所有可能值之和。 （4 分）

A 12-sided polygon possesses rotational symmetry of order k, where $1<k<12$. Find the sum of all possible values of k. (4 marks)

8. 在飯店乙，每客午餐的標價比飯店甲高 $n\%$。可是飯店甲設加一服務費，飯店乙不設加一服務費之餘還有八五折優惠，結果兩所飯店每客午餐的實際收費相同。求最接近 n 的整數。 （4分）

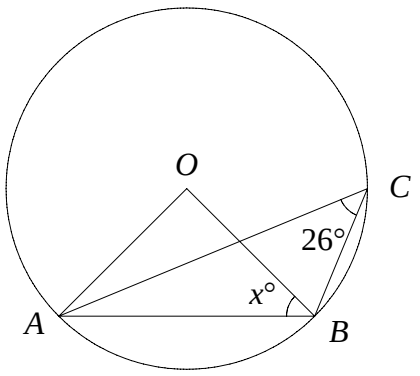
In Restaurant B, the marked price of each set lunch is $n\%$ higher than that in Restaurant A. However, Restaurant A charges a 10% service charge, while Restaurant B does not charge any service charge and provides a 15% discount. It then turns out that the actual price of a set lunch is the same in both restaurants. Find the integer closest to n. (4 marks)

9. 陳先生從甲城駕車往乙城，兩地相距 n 公里。若他把速率由每秒 25 米提高至每小時 100 公里，則所需時間會減少 1 小時。求 n。 （4分）

Mr Chan drives from City A to City B which are n km apart. If he raises the speed from 25 m/s to 100 km/h, the time taken will be reduced by 1 hour. Find n. (4 marks)

10. 圖中，O 是圓心，A、B、C 是圓周上的三點。若 $\angle ACB = 26°$、$\angle OBA = x°$，求 x。 （5分）

In the figure, O is the centre of the circle and A, B, C are three points on the circumference. If $\angle ACB = 26°$ and $\angle OBA = x°$, find x. (5 marks)



11. 一個非長方形的四邊形的四隻內角分別爲 $p°$、$q°$、$r°$ 和 $s°$，其中 p、q、r、s 爲正整數，且它們的最大公因數爲 d。求 d 的最大可能值。 （5分）

A non-rectangular quadrilateral has interior angles $p°$, $q°$, $r°$ and $s°$, where p, q, r, s are positive integers with H.C.F. d. Find the greatest possible value of d. (5 marks)

12. 在所示的加法中，每個字母代表一個 0 至 9 的不同數字。求 FOUR 所代表的四位數的最大可能值。 （6分）

In the addition shown, each letter represents a different integer from 0 to 9. Find the greatest possible value of the four-digit number represented by FOUR. (6 marks)

$$\begin{array}{rcccccc} & & F & O & U & R \\ + & & F & O & U & R \\ \hline & E & I & G & H & T \end{array}$$

13. 某袋子中有 m 個紅球和 n 個藍球，其中 $m+n=100$。每個球上都有一個不超過 100 的正整數，沒有兩個球上的整數相同，且所有紅球上的整數之和等於所有藍球上的整數之和。求 mn 的最小可能值。（6 分）

In a bag there are m red balls and n blue balls, where $m+n=100$. On each ball there is a positive integer not exceeding 100, the numbers on the balls are pairwise distinct and the sum of the numbers on all red balls is equal to the sum of the numbers on all blue balls. Find the smallest possible value of mn. (6 marks)

14. 設 $[x]$ 代表不超過 x 的最大整數，例如 [1.1] = 1、[6.9] = 6 和 [5] = 5。求 $\left[\sqrt{1\times5+20}\right]+\left[\sqrt{2\times6+20}\right]+\cdots+\left[\sqrt{100\times104+20}\right]$ 的值。（6 分）

Let $[x]$ denote the greatest integer not exceeding x. For example, [1.1] = 1, [6.9] = 6 and [5] = 5. Find the value of $\left[\sqrt{1\times5+20}\right]+\left[\sqrt{2\times6+20}\right]+\cdots+\left[\sqrt{100\times104+20}\right]$. (6 marks)

15. 美英打算用六支分別長 3、7、8、8、9 和 12 的棒作稜，砌成一個四面體。她的四面體可以有多少個不同的形狀？（如果兩個四面體經適當的反射和旋轉後重疊，我們視它們爲相同的形狀。）（6 分）

Ann wants to construct a tetrahedron with six sticks as edges, whose lengths are 3, 7, 8, 8, 9 and 12. How many different possible shapes of the tetrahedron are there? (If two tetrahedra coincide after suitable reflection and rotation, we regard them to be the same shape.) (6 marks)

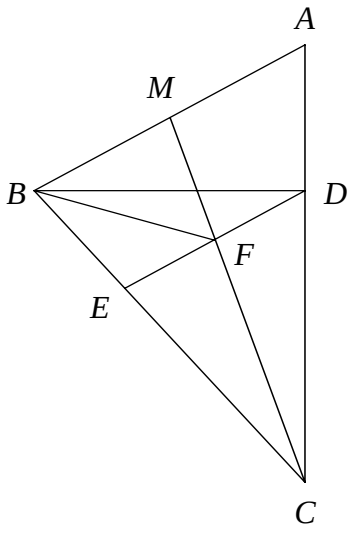
16. 俊輝的手錶能顯示每天的日期中的「日」。手錶的日期盤上順時針順序寫著 1 至 31 這 31 個不同的整數，每天凌晨盤上的指針都會向順時針方向跳向下一個數字。因爲有些月份不足 31 天，所以有些時候指針需要人手調較。可是俊輝從來沒調較過。已知在 2008 年 2 月，俊輝的手錶顯示著正確的「日」。下一次手錶再顯示正確的日期是多少個月後的事？（6 分）

Dan's watch can show the 'day' of the date. On the date plate of the watch, the 31 different integers 1 to 31 are written in clockwise order. The pointer on the plate springs to the next number in the clockwise direction at midnight every day. As there are fewer than 31 days in some months, the pointer has to be adjusted at times. However, Dan never adjusts it. It is known that his watch shows the correct 'day' in February 2008. How many months later will his watch show the correct 'day' again? (6 marks)

17. 一個直立的圓柱形量筒盛了高 n cm 的水，其中 $n>12$。已知如果在量筒上開洞，只要水面仍在洞之上時，水便會自該洞以均速流走，並且每個洞的水流速度都相等。當在筒底、高 5 cm 和高 12 cm 處各開一個洞，則水會在 5 分鐘後全部流走。當在筒底、高 5 cm、高 9 cm 和高 12 cm 處各開一個洞，則水會在 4 分鐘全部流走。求 n 的值。（7 分）

An upright cylindrical measuring cylinder is filled with water to n cm high, where $n>12$. It is known that if holes are drilled on the cylinder, water will flow away from the hole at a constant rate as long as the water level is above the hole. The flow rates in all holes are equal. When one hole is drilled each at the base, 5 cm high and 12 cm high, all water will flow away in 5 minutes. When one hole is drilled each at the base, 5 cm high, 9 cm high and 12 cm high, all water will flow away in 4 minutes. Find the value of n. (7 marks)

18. 圖中，$\angle ACB = 15°$、D 是 B 到 AC 的垂足、M 是 AB 的中點。過 D 作平行於 AB 的直線交 BC 於 E，且 CM 交 DE 於 F。若 $FM = FB$，且 $\angle ABC = x°$，求 x。（7 分）



In the figure, $\angle ACB = 15°$, D is the foot of the perpendicular from B to AC and M is the mid-point of AB. Through D a line parallel to AB is constructed to meet BC at E, and CM meets DE at F. If $FM = FB$ and $\angle ABC = x°$, find x. (7 marks)

19. 設 $a_1 = k$（其中 k 是小於 100 的正整數），且對所有正整數 n 皆有 $a_{n+1} = a_n^{a_n}$。若數列 a_1、a_2、a_3、⋯ 中沒有兩個連續項的個位數字相同，求 k 所有可能值之和。（7 分）

Let $a_1 = k$ (where k is a positive integer less than 100) and $a_{n+1} = a_n^{a_n}$ for all positive integers n. If the sequence a_1, a_2, a_3, … does not consist of two consecutive terms with the same unit digit, find the sum of all possible values of k. (7 marks)

20. 某村莊有四名警察。他們分別每 3 天、4 天、11 天和 37 天到村莊巡邏一次。最多有連續多少天村莊都有警察巡邏？ （7 分）

There are four policemen in a village. They patrol in the village once every 3, 4, 11 and 37 days respectively. What is the maximum number of consecutive days in which there is policeman patrolling in the village? (7 marks)

全卷完

END OF PAPER