第七屆培正數學邀請賽

7th Pui Ching Invitational Mathematics Competition

初賽（中二組）

Heat Event (Secondary 2)

時限：1 小時 15 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

1. 本卷共設 20 題，總分爲 100 分。

   There are 20 questions in this paper and the total score is 100.

2. 除特別指明外，本卷內的所有數均爲十進制。

   Unless otherwise stated, all numbers in this paper are in decimal system.

3. 所有答案皆是 0 至 9999 之間的整數（包括 0 和 9999）。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

   All answers are integers between 0 and 9999 (including 0 and 9999). Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

4. 不得使用計算機。

   The use of calculators is not allowed.

5. 本卷的附圖不一定依比例繪成。

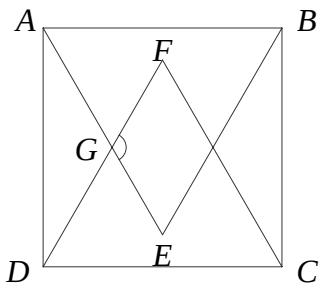
   The diagrams in this paper are not necessarily drawn to scale.

1. 若 n 是兩位數，而 20080126 是 n 的倍數，求 n 的最小可能值。 （3分）

If n is a two-digit number and 20080126 is a multiple of n, find the smallest possible value of n. (3 marks)

2. 圖中，$ABCD$ 是正方形，ABE 和 CDF 是等邊三角形。AE 與 DF 交於 G。若 $\angle EGF = x°$，求 x。 （3分）

In the figure, $ABCD$ is a square while ABE and CDF are equilateral triangles. AE and DF meet at G. If $\angle EGF = x°$, find x. (3 marks)



3. 求 9998×9998 的最後四位數字。 （3分）

Find the last four digits of 9998×9998. (3 marks)

4. 在首 2008 個正整數中，哪一個的數字之和最大？ （3分）

Among the first 2008 positive integers, which one has the greatest sum of digits? (3 marks)

5. ABC 是直角三角形，它的每隻內角以「度」表示時都是整數。若 $\angle A - \angle B = x°$，問 x 有多少個可能值？ （4分）

ABC is a right-angled triangle with all interior angles (in degrees) being integers. If $\angle A - \angle B = x°$, how many possible values of x are there? (4 marks)

6. 求 $0-2+4-6+8-10+\cdots-2006+2008$ 的值。 （4分）

Find the value of $0-2+4-6+8-10+\cdots-2006+2008$. (4 marks)

7. 某次聚會有 50 名男孩和 n 名女孩參加，當中某些男孩和女孩曾經互相握手。若每名男孩和最少 5 名女孩握手，而每名女孩和最多 3 名男孩握手，求 n 的最小可能值。 （5分）

In a party there are 50 boys and n girls. Some boys and girls have shaken hands with each other. If each boy shakes hands with at least 5 girls while each girl shakes hands with at most 3 boys, find the smallest possible value of n. (5 marks)

8. 某長方形的長增加 n%、闊減少 n% 後，面積減少了 15%。求最接近 n 的整數。（5分）

When the length of a rectangle is increased by n% and its width is decreased by n%, its area is decreased by 15%. Find the integer closest to n. (5 marks)

9. 小華做方程的練習題時，發現作業簿上其中兩道題的某些數字被塗污了（見附圖）。當他從書後翻查這兩道題的答案時，他發現兩題的答案是相同的。他並由此推斷出被塗污的是兩個相同的整數。求此整數。（5分）

When working on exercise on equations, Roy found that some figures have been defaced in two equations (see the figure). He then checked the answers at the end of the book and found that the answers to both questions are the same. From this he deduced that the same integer was being defaced in both equations. Find this integer. (5 marks)

(a) $2x - \blacksquare = x - 2$

(b) $\dfrac{x-4}{3} = \blacksquare - (x+1)$

10. 小嘉把首 n 個正方形數加起來（即 $1^2 + 2^2 + 3^2 + \cdots + n^2$），並發現答案的個位數字是 k。求 k 所有可能值之和。（5分）

Chris added up the first n square numbers (i.e. $1^2 + 2^2 + 3^2 + \cdots + n^2$) and found that the answer has unit digit k. Find the sum of all possible values of k. (5 marks)

11. 朝偉在 2006 年發現情人節（2 月 14 日）和白色情人節（3 月 14 日）都在星期二，於是他以爲每年的這兩個節日都在星期的同一天。在 2007 年及以後每年的情人節，當他知道當日是星期幾後，便跟著說當年的白色情人節也是星期幾。直到 2046 年過後，家衛才告訴他這並不是一定正確的。問朝偉在 2007 至 2046 年間（包括首尾兩年），說對了多少次？（5分）

In 2006, Tony found that both the Valentine's Day (14th February) and the White Day (14th March) were Tuesdays. He thus thought that these two days fall on the same day of the week every year. In 2007 and each subsequent year, once he knows the day of the week of the Valentine's Day, he makes a statement that the White Day that year is on the same day of the week. It is not until after 2046 when Kar-wai tells him that this is not necessarily true. For how many times is Tony correct between the years 2007 and 2046 (inclusive)? (5 marks)

12. 某書店有七本新書出售，每本的售價（以「元」爲單位）都是正整數。七本新書從左至右排列時，最左邊的三本共值 20 元、中間的三本共值 30 元、最右邊的三本則共值 40 元。問最多有幾本書的售價是 10 元？ （5 分）

In a bookstore seven new books are on sale. The price of each book (in dollars) is a positive integer. When the seven books are lined up from left to right, the total price of the three books on the left is 20 dollars, the total price of the three in the middle is 30 dollars while the total price of the three on the right is 40 dollars. What is the maximum number of books that may be worth 10 dollars? (5 marks)

13. 某數學比賽以隊際形式進行，每隊規定由同一學校的 6 或 7 名學生組成。某學校派了 n 名學生參加這個比賽，並發現他們最少需組成 31 隊，最多可組成 36 隊。求 n 所有可能值之和。 （6 分）

In a mathematical competition, participants are required to form teams. Each team must consist of 6 or 7 students from the same school. A school sent n students for the competition, and they found that they must form at least 31 teams and at most 36 teams. Find the sum of all possible values of n. (6 marks)

14. 小明寫下了 10 個正整數，它們之和是 2008，而它們的最大公因數爲 d。求 d 的最大可能值。 （6 分）

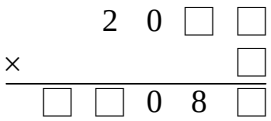
Mike wrote down 10 positive integers with sum 2008, and the H.C.F. of these integers is d. Find the greatest possible value of d. (6 marks)

15. 小明和小強在圓周爲 8 km 的圓形單車徑上踏單車。他們同時於同一點分別向順時針方向和逆時針方向出發，每當他們相遇後便立即折返，駛向與原來相反的方向，但速度減半。已知小明和小強開始時的速率分別爲 25 km/h 和 15 km/h，求出發後首 3 小時小明駛過的距離（以 km 爲單位）。 （6 分）

Sam and Tom cycled on a circular bicycle track with circumference 8 km. They started their journey in the clockwise and anti-clockwise directions respectively at the same point and same time. Whenever they met, they turned back and rode in the opposite direction immediately with their speeds reduced by half. It is known that the initial speeds of Sam and Tom were 25 km/h and 15 km/h respectively. Find the distance (in km) Sam had ridden in the first 3 hours. (6 marks)

16. 圖中顯示一條乘式，但當中有些數字留空了。求積（即最底一行的五位數）最大與最小可能值之差。（6分）

The figure shows a multiplication, but some digits are left out. Find the difference between the maximum and minimum values of the product (i.e. the five-digit number in the bottom row). (6 marks)



17. 小美有五角、一元和二元硬幣各 10 個。她有多少種方法付款剛好 10 元？（6分）

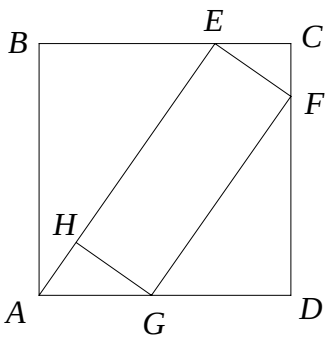
Mimi has 10 fifty-cent, 10 one-dollar and 10 two-dollar coins. In how many ways can she pay exactly 10 dollars? (6 marks)

18. 平面上有三個半徑分別爲 1、2、3 的圓，它們當中任意兩個的圓心距離均爲 10。若要在平面上加上第四個圓 C，使得 C 和原來的三個圓都有剛好一個交點，問 C 的位置有多少個不同的可能性？（6分）

On the plane there are three circles with radii 1, 2, 3 respectively and the distance between the centres of any two of them is 10. If a fourth circle C is to be added on the plane such that C meets each of the three original circles at exactly one point, how many choices of the position of C are there? (6 marks)

19. 圖中，$ABCD$ 是正方形，E、F、G、H 分別爲 BC、CD、DA 和 AE 上的點，使得 $EFGH$ 爲長方形。已知 $BE=3$、$AB=4$、$AE=5$。若 $EFGH$ 的面積爲 $\frac{m}{n}$，其中 m 和 n 爲正整數且它們的最大公因數爲 1，求 $m+n$。（7分）

In the figure, $ABCD$ is a square. E, F, G, H are points on BC, CD, DA and AE respectively such that $EFGH$ is a rectangle. It is known that $BE=3$, $AB=4$ and $AE=5$. If $EFGH$ has area $\frac{m}{n}$ where m, n are positive integers with H.C.F. 1, find $m+n$. (7 marks)



20. 某項比賽共有 7 名裁判 A、B、C、D、E、F 和 G。裁判評分後，當中最高和最低的各一個分數會被刪去，餘下的五個分數之和即爲參賽者的分數。例如，如果七位裁判分別給 3、3、3、4、6、7、7 分時，參賽者的分數則爲 $3+3+4+6+7$ 分，即 23 分。子賢參加了該項比賽，並知道下列的事實：

如果裁判 A 多給他 2 分，則他的分數會多 1 分。
如果裁判 A 少給他 3 分，則他的分數會少 2 分。
無論裁判 B 多給他 3 分或少給他 1 分，他的分數都不會改變。
如果裁判 C 多給他 2 分，則他的分數不會改變。
裁判 C 給了他 4 分。

求子賢的分數。（7 分）

Scores are given by 7 judges A, B, C, D, E, F and G in a competition. When scores are given, one highest score and one lowest score are discarded. The sum of the five remaining scores is the score of the contestant. For example, if the 7 judges give 3, 3, 3, 4, 6, 7, 7 marks, the score of the contestant is $3+3+4+6+7$ marks, i.e. 23 marks. Leo joined the competition and he knew the following facts:

If Judge A gave him 2 marks more, his score would increase by 1.
If Judge A gave him 3 marks fewer, his score would decrease by 2.
If Judge B gave him 3 marks more or 1 mark fewer, his score would remain unchanged.
If Judge C gave him 2 marks more, his score would remain unchanged.
Judge C gave him 4 marks.

Find Leo's score. (7 marks)

全卷完

END OF PAPER