

第七屆培正數學邀請賽

7th Pui Ching Invitational Mathematics Competition

初賽（中一組）

Heat Event (Secondary 1)

時限：1 小時 15 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

1. 本卷共設 20 題，總分爲 100 分。

There are 20 questions in this paper and the total score is 100.

2. 除特別指明外，本卷內的所有數均爲十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

3. 所有答案皆是 0 至 9999 之間的整數（包括 0 和 9999）。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

All answers are integers between 0 and 9999 (including 0 and 9999). Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

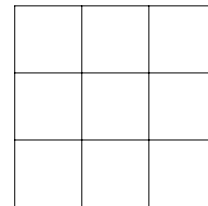
4. 不得使用計算機。

The use of calculators is not allowed.

5. 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

1. 現要在圖中的 3×3 表格的九格中把其中三格塗上顏色，使得塗了色的三個方格的中心成一直線。問有多少種方法選三個方格來塗色？



(3 分)

The figure shows a 3×3 table and three of the nine cells are to be coloured so that the centres of the three coloured cells lie on the same straight line. In how many ways can we choose three cells to be coloured?

(3 marks)

2. 若 n 是兩位數，而 20080126 是 n 的倍數，求 n 的最小可能值。

(3 分)

If n is a two-digit number and 20080126 is a multiple of n , find the smallest possible value of n .

(3 marks)

3. 求 $12345 + 23456 + 34567 + 45678 + 56789 + 67890 + 78901 + 89012 + 90123$ 的最後四位數字。

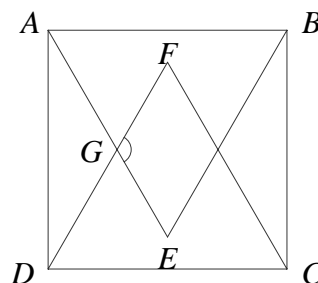
(3 分)

Find the last four digits of

$$12345 + 23456 + 34567 + 45678 + 56789 + 67890 + 78901 + 89012 + 90123.$$

(3 marks)

4. 圖中， $ABCD$ 是正方形， ABE 和 CDF 是等邊三角形。 AE 與 DF 交於 G 。若 $\angle EGF = x^\circ$ ，求 x 。



(3 分)

In the figure, $ABCD$ is a square while ABE and CDF are equilateral triangles. AE and DF meet at G . If $\angle EGF = x^\circ$, find x .

(3 marks)

5. 求最小的正整數 n ，使得 $\sqrt{7n+4}$ 為平方數。

(4 分)

Find the smallest positive integer n for which $\sqrt{7n+4}$ is a square number.

(4 marks)

6. 某次聚會有 50 名男孩和 n 名女孩參加，當中某些男孩和女孩曾經互相握手。若每名男孩和最少 5 名女孩握手，而每名女孩和最多 3 名男孩握手，求 n 的最小可能值。

(5 分)

In a party there are 50 boys and n girls. Some boys and girls have shaken hands with each other. If each boy shakes hands with at least 5 girls while each girl shakes hands with at most 3 boys, find the smallest possible value of n .

(5 marks)

7. 陳太太打算在百貨公司買一條標價 n 元的裙子。公司正值大減價，所有貨品均以標價八折出售。陳太太同時持有貴賓咭，可額外再享九折優惠。付款時，職員告知陳太太貴賓咭的「額外九折」已改為「額外再減 20 元」。之後陳太太發現，這項改變對她實際需付的金額並沒有影響。求 n 。(5 分)

Mrs Chan plans to buy a dress marked at n dollars at a department store. The department store is having a sales promotion in which all goods are sold 20% off the marked price. Being a VIP card holder, Mrs Chan is entitled to an extra 10% off. However, when queuing for the cashier, Mrs Chan was told that the 'extra 10% off' has been changed to 'extra \$20 discount'. Mrs Chan then found that such change does not affect the actual amount she has to pay. Find n . (5 marks)

8. 小嘉把首 n 個正整數加起來（即 $1+2+3+\cdots+n$ ），並發現答案的個位數字是 k 。求 k 所有可能值之和。(5 分)

Chris added up the first n positive integers (i.e. $1+2+3+\cdots+n$) and found that the answer has unit digit k . Find the sum of all possible values of k . (5 marks)

9. 某書店有七本新書出售，每本的售價（以「元」為單位）都是正整數。七本新書從左至右排列時，最左邊的三本共值 20 元、中間的三本共值 30 元、最右邊的三本則共值 40 元。問最多有幾本書的售價是 10 元？(5 分)

In a bookstore seven new books are on sale. The price of each book (in dollars) is a positive integer. When the seven books are lined up from left to right, the total price of the three books on the left is 20 dollars, the total price of the three in the middle is 30 dollars while the total price of the three on the right is 40 dollars. What is the maximum number of books that may be worth 10 dollars? (5 marks)

10. 某數學比賽以隊際形式進行，每隊規定由同一學校的 6 或 7 名學生組成。甲、乙兩所學校分別派了 n 名學生參加這個比賽，其中甲校的學生組成了 32 隊、乙校的學生組成了 36 隊。求 n 的最小可能值。(5 分)

In a mathematical competition, participants are required to form teams. Each team must consist of 6 or 7 students from the same school. Schools A and B has each sent n students for the competition. The students from School A are divided into 32 teams while the students from School B are divided into 36 teams. Find the smallest possible value of n . (5 marks)

11. 已知 m 和 n 為正整數，且它們的最小公倍數為 20。問 m 有多少個不同的可能值？ (5 分)

Given m, n are positive integers and their L.C.M. is 20. How many different possible values of m are there? (5 marks)

12. 某項比賽共有 7 名裁判評分。每位評判的分數皆為 0 至 10 之間的整數（包括 0 和 10）。裁判評分後，當中最高和最低的各一個分數會被刪去，餘下的五個分數之和即為參賽者的分數。例如，如果七位裁判分別給 3、3、3、4、6、7、7 分時，參賽者的分數則為 $3+3+4+6+7$ 分，即 23 分。敏光 and 家昇參加了該項比賽。已知首 6 位裁判對敏光的評分的總和是 32，而首 6 位裁判對家昇的評分的總和則為 41。可是當第 7 位裁判評分後，敏光的分數卻比家昇高。求家昇的分數。 (5 分)

Scores are given by 7 judges in a competition. The score given by each judge is an integer between 0 and 10 (inclusive). When scores are given, one highest score and one lowest score are discarded. The sum of the five remaining scores is the score of the contestant. For example, if the 7 judges give 3, 3, 3, 4, 6, 7, 7 marks, the score of the contestant is $3+3+4+6+7$ marks, i.e. 23 marks. Matt and Nelson joined the competition. It is known that the sum of scores from the first 6 judges for Matt was 32, while the sum of scores from the first 6 judges for Nelson was 41. However, Matt's score turned out to be higher than that of Nelson after the 7th judge gave the scores. Find Nelson's score. (5 marks)

13. 圖中顯示一條乘式，但當中有些數字留空了。求積（即最底一行的五位數）最大與最小可能值之差。

The figure shows a multiplication, but some digits are left out. Find the difference between the maximum and minimum values of the product (i.e. the five-digit number in the bottom row).

$$\begin{array}{r} 20\Box\Box \\ \times \Box\Box08\Box \\ \hline \end{array}$$

(6 分)

(6 marks)

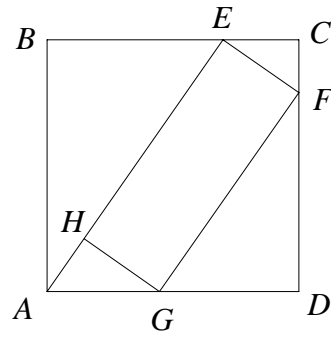
14. 某三角形的三隻內角分別是 x° 、 y° 和 z° ，其中 x 、 y 、 z 都是質數且 $x < y < z$ 。求 $z - y$ 的最小可能值。 (6 分)

The three interior angles of a triangle are x° , y° and z° , where x, y, z are prime numbers with $x < y < z$. Find the smallest possible value of $z - y$. (6 marks)

15. 小美有五角、一元和二元硬幣各 10 個。她有多少種方法付款剛好 10 元？ (6 分)
- Mimi has 10 fifty-cent, 10 one-dollar and 10 two-dollar coins. In how many ways can she pay exactly 10 dollars? (6 marks)
16. 2008 名學生由左至右排成一列。從左邊數起的每第 13 位學生獲贈一顆糖果，而從右邊數起的每第 17 位學生則獲贈一枝鉛筆。有多少名學生同時得到糖果和鉛筆？ (6 分)
- 2008 students line up in a row from left to right. Starting from the left, every 13th student is given a piece of candy; starting from the right, every 17th student is given a pencil. How many students get both the candy and the pencil? (6 marks)
17. 小明和小強在圓周為 6 km 的圓形單車徑上踏單車。他們同時於同一點分別向順時針方向和逆時針方向出發，每當他們相遇後便立即折返，駛向與原來相反的方向，但速度減半。已知小明和小強開始時的速率分別為 23 km/h 和 17 km/h，求二人第 20 次重遇前小明駛過的距離（以 km 為單位）。 (6 分)
- Sam and Tom cycled on a circular bicycle track with circumference 6 km. They started their journey in the clockwise and anti-clockwise directions respectively at the same point and same time. Whenever they met, they turned back and rode in the opposite direction immediately with their speeds reduced by half. It is known that the initial speeds of Sam and Tom were 23 km/h and 17 km/h respectively. Find the distance (in km) that Sam had ridden before their 20th meeting. (6 marks)
18. 已知 a 、 b 、 c 是正整數，其中 $a < b < c$ 且 $c^3 - b^3 < b^3 - a^3$ 。求 $a + b + c$ 的最小可能值。 (6 分)
- Let a, b, c be positive integers such that $a < b < c$ and $c^3 - b^3 < b^3 - a^3$. Find the smallest possible value of $a + b + c$. (6 marks)
19. 平面上有三個半徑分別為 1、2、3 的圓，它們當中任意兩個的圓心距離均為 10。若要在平面上加上第四個圓 C ，使得 C 和原來的三個圓都有剛好一個交點，問 C 的位置有多少個不同的可能性？ (6 分)
- On the plane there are three circles with radii 1, 2, 3 respectively and the distance between the centres of any two of them is 10. If a fourth circle C is to be added on the plane such that C meets each of the three original circles at exactly one point, how many choices of the position of C are there? (6 marks)

20. 圖中， $ABCD$ 是正方形， E 、 F 、 G 、 H 分別為 BC 、 CD 、 DA 和 AE 上的點，使得 $EFGH$ 為長方形。已知 $BE=3$ 、 $AB=4$ 、 $AE=5$ 。若 $EFGH$ 的面積為 $\frac{m}{n}$ ，其中 m 和 n 為正整數且它們的最大公因數為 1，求 $m+n$ 。

In the figure, $ABCD$ is a square. E , F , G , H are points on BC , CD , DA and AE respectively such that $EFGH$ is a rectangle. It is known that $BE=3$, $AB=4$ and $AE=5$. If $EFGH$ has area $\frac{m}{n}$ where m , n are positive integers with H.C.F. 1, find $m+n$.



(7 分)

(7 marks)

全卷完

END OF PAPER