第六屆培正數學邀請賽

6th Pui Ching Invitational Mathematics Competition

初賽（高中組）

Heat Event (Senior Secondary)

時限：1 小時 15 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

1. 本卷共設 20 題，總分爲 100 分。

   There are 20 questions in this paper and the total score is 100.

2. 除特別指明外，本卷內的所有數均爲十進制。

   Unless otherwise stated, all numbers in this paper are in decimal system.

3. 所有答案皆是 0 至 9999 之間的整數（包括 0 和 9999）。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

   All answers are integers between 0 and 9999 (including 0 and 9999). Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

4. 不得使用計算機。

   The use of calculators is not allowed.

5. 本卷的附圖不一定依比例繪成。

   The diagrams in this paper are not necessarily drawn to scale.

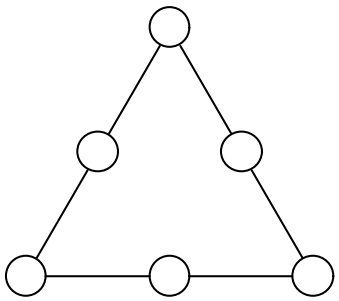
1. 小斌把所有小於 1000000、個位數字不是 0 而且是 2007 的倍數的正整數乘起來。乘積的最後四位數字是甚麼？ （3 分）

Purple multiplied together all positive integers less than 1000000 which are multiples of 2007 with unit digit non-zero. What are the last four digits of the product? (3 marks)

2. 當 $x^{2007}+k$ 除以 $x+1$ 時，餘數是 127。求 k。 （3 分）

When $x^{2007}+k$ is divided by $x+1$, the remainder is 127. Find k. (3 marks)

3. 現要在圖中的六個圓中分別填上 1 至 6，並計算每條邊上三個圓內的數字之和。這三個數字之和加起來的最小值是甚麼？ （3 分）



The numbers 1 to 6 are to be filled into the six circles in the figure. The sum of the three numbers in the three circles on each side is then computed and the three sums are added up. What is the smallest possible value of the result obtained? (3 marks)

4. 一個等差數列的首項 a 爲整數，公差爲 -5。若第 127 項是負數，求 a 的最大可能值。 （4 分）

An arithmetic sequence has first term a which is an integer and common difference -5. If the 127th term is negative, find the greatest possible value of a. (4 marks)

5. 一條直線的 x 截距和 y 截距分別爲 a 和 b。已知 a、b 兩數均非零，且其中一個的絕對值是另一個的絕對值的三倍。那麼該直線的斜率有多少個不同的可能值？ （4 分）

A straight line has x-intercept a and y-intercept b. It is known that both a and b are non-zero, and the absolute value of one is three times the absolute value of the other. How many different possible values of the slope of the straight line are there? (4 marks)

6. 若圓 $x^2+y^2-4x+8y+k=0$ 的半徑是正整數，求 k 的最大可能值。 （4 分）

If the radius of the circle $x^2+y^2-4x+8y+k=0$ is a positive integer, find the greatest possible value of k. (4 marks)

7. 林先生每天下班後都從辦公室駕車回家。一天，他把駕車的速度提高了 40%，結果全程所需的時間比平日減少了 $m\%$。求最接近 m 的整數。（4 分）

Every day, Mr Lam drives home from office after work. One day, he increased the speed by 40% and the time taken was reduced by $m\%$ as a result. Find the integer closest to m. (4 marks)

8. 設 $[x]$ 代表不超過 x 的最大整數，例如 $[1.1] = 1$、$[6.9] = 6$ 和 $[5] = 5$。求 $[\log_2 100]+[\log_3 100]+[\log_4 100]+\cdots+[\log_{99} 100]$ 的值。（4 分）

Let $[x]$ denote the greatest integer not exceeding x. For example, $[1.1] = 1$, $[6.9] = 6$ and $[5] = 5$. Find the value of $[\log_2 100]+[\log_3 100]+[\log_4 100]+\cdots+[\log_{99} 100]$. (4 marks)

9. 已知方程 $x(x-5)(x-12)(x-13)=900$ 只有一個正實根，求該正實根。（5 分）

Given that the equation $x(x-5)(x-12)(x-13)=900$ has only one positive real root, find that positive real root. (5 marks)

10. 某城市有兩條巴士線，一條來往碼頭和學校，另一條來往碼頭和市集。從碼頭到學校（或學校到碼頭）的車程爲 40 分鐘，而從碼頭到市集（或市集到碼頭）的車程爲 15 分鐘。若兩線均需維持 25 分鐘一班車，則最少共要派出多少輛巴士行走？（5 分）

In a city there are two bus routes. One goes to and from the pier and the school, while the other goes to and from the pier and the market. The journey from the pier to the school (or from the school to the pier) takes 40 minutes while the journey from the pier to the market (or from the market to the pier) takes 15 minutes. If the frequencies of both routes have to be maintained at 25 minutes, at least how many buses are needed? (5 marks)

11. 設 k 爲正整數。若方程組 $\begin{cases}2x+3y-6=0\\3x-2y+k=0\end{cases}$ 中 x 和 y 的解都是整數，求 k 的最小可能值。（5 分）

Let k be a positive integer. If the system of equations $\begin{cases}2x+3y-6=0\\3x-2y+k=0\end{cases}$ has integer solutions in x and y, find the smallest possible value of k. (5 marks)

12. 從 1、2、…、100 中，最少要選出多少個整數，才能確保當中必定有兩個之積的個位數字是 0？ （5 分）

What is the least number of integers that must be chosen from 1, 2, …, 100 to ensure that we must be able to find two of the chosen integers whose product has unit digit 0? (5 marks)

13. 一個主題公園的成人、長者和小童入場券分別價值 295 元、170 元和 210 元。每次購買剛好 10 張入場券時，價值最高的一張可獲免費。若某人需要 7 張成人、7 張長者和 5 張小童入場券，他最少要付款多少元？ （5 分）

In a theme park, a ticket for adults, elderly and children cost $295, $170 and $210 respectively. For a one-time purchase of exactly 10 tickets, the highest-valued one will be free of charge. If someone needs 7 tickets for adults, 7 tickets for elderly and 5 tickets for children, what is the minimum amount (in dollars) that he has to pay? (5 marks)

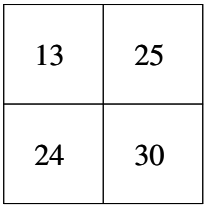
14. 若某天的「月」和「日」之積是合成數，則那天稱爲「好日子」。例如：因爲 $1\times27=27$，而 27 是合成數，所以 1 月 27 日是「好日子」。那麼，最多可以有連續多少天的「好日子」？ （6 分）

A day is said to be ‘good’ if the product of the ‘month’ and the ‘day’ is a composite number. For example, since $1\times27=27$ and 27 is a composite number, we say that 27th January is a ‘good day’. What is the maximum possible number of consecutive ‘good days’? (6 marks)

15. 在一個三角形中，其中兩條邊的長度相同，而且都比第三條邊短。三邊的長度都是整數，而且都是方程 $x^2-127x+k=0$（其中 k 是常數）的根。求 k 的最小可能值。 （6 分）

In a triangle, two sides have equal lengths and are both shorter than the third side. The lengths of the three sides are all integers and all satisfy the equation $x^2-127x+k=0$, where k is a constant. Find the smallest possible value of k. (6 marks)

16. 如圖所示，某個國家的地形是正方形，並分成四個區域。該國有一些公園，每個都位於一個區域內或橫跨兩個相鄰（即圖中有公共邊）的區域。每個區域中的整數表示完全或部分位於該區域內的公園的數目。問該國家最少有多少個公園？（6 分）



A country is in the shape of a square and is divided into four regions as shown. There are some parks in the country and each park lies in one region or across two neighbouring regions (i.e. regions with a common edge in the figure). The integer in each region denotes the number of parks which lie wholly or partly in the region. What is the minimum total number of parks in the country? (6 marks)

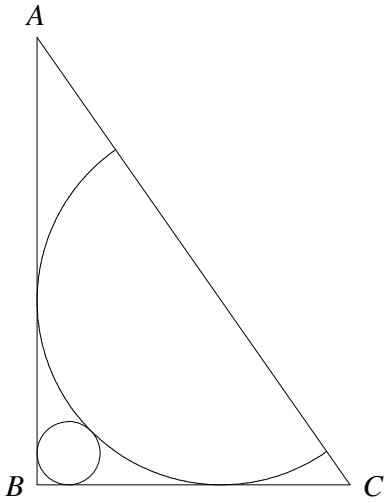
17. 小琪把所有不超過 2007 而且當中包含數字「2」的正整數順序寫下來，得到一個新的整數 21220212223…20062007。這個整數中有多少個「0」？（7 分）

Angela formed a positive integer by writing down in ascending order all positive integers not exceeding 2007 which contain the digit '2', i.e. 21220212223…20062007. How many '0's are there in this integer? (7 marks)

18. 某城市規定，市民購物時須繳交相當於貨品原價 5% 的銷售稅。王先生購物後，發現貨品的原價和連稅的價格（以「元」爲單位）都是整數。它們的數字都相同，只是數字的次序改變了。已知貨品的原價介乎 1000 元和 4000 元之間，則貨品的原價是多少元？（7 分）

In a city, citizens are required to pay 5% of the original price of goods as sales tax during purchase. After purchase, Mr Wong discovered that both the original price and the price after tax of the goods (in dollars) are integers. They have the same digits but in different orders. Given that the original price of the goods is between 1000 dollars and 4000 dollars, find the original price of the goods in dollars. (7 marks)

19. 圖中，ABC 是直角三角形，B 是直角，而且 $AB = 28$ 、 $BC = 21$ 。半圓的直徑位於 AC 上，而半圓分別與 AB 和 BC 相切。小圓則分別與 AB、BC 和半圓相切。已知小圓的半徑可寫成 $a + b\sqrt{2}$ ，其中 a 和 b 是整數。求 $a^2 + b^2$ 的值。 （7分）



In the figure, ΔABC is right-angled at B with $AB = 28$ and $BC = 21$. The semicircle has its diameter lying on AC and is tangent to both AB and BC. The small circle is tangent to AB, BC and the semicircle. It is known that the radius of the small circle can be written in the form $a + b\sqrt{2}$, where a and b are integers. Find the value of $a^2 + b^2$. (7 marks)

20. 在所示的加法中，每個字母代表一個 1 至 9 的不同數字。求 DEFG 所代表的四位數。 （7分）

In the addition shown, each letter represents a different integer from 1 to 9. Find the four-digit number represented by DEFG. (7 marks)

$$\begin{array}{rcccc} & A & B & C & \\ & A & C & B & \\ & B & A & C & \\ & B & C & A & \\ & C & A & B & \\ + & C & B & A & \\ \hline & D & E & F & G \end{array}$$

全卷完

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