初賽（中四組）

Heat Event (Secondary 4)

時限：**1 小時 15 分**

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

1. 本卷共設 20 題，總分為 100 分。

    There are 20 questions in this paper and the total score is 100.

2. 除特別指明外，本卷內的所有數均為十進制。

    Unless otherwise stated, all numbers in this paper are in decimal system.

3. 所有答案皆是 0 至 9999 之間的整數（包括 0 和 9999）。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

    All answers are integers between 0 and 9999 (including 0 and 9999). Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

4. 不得使用計算機。

    The use of calculators is not allowed.
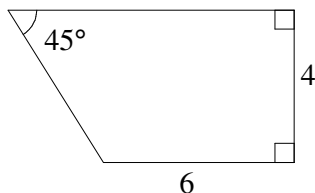
5. 本卷的附圖不一定依比例繪成。

    The diagrams in this paper are not necessarily drawn to scale.

1. 當 123456789 除以 18 時，餘數是多少？ （2分）

   What is the remainder when 123456789 is divided by 18? (2 marks)

2. 求圖中的四邊形的面積。（如有需要，答案
   準確至最接近整數。） （3分）

   Find the area of the quadrilateral in the figure.
   (Round off the answer to the nearest integer if
   necessary.) (3 marks)

3. 從 1、2、…、100 中，最少要選出多少個整數，才能確保當中必定有兩個的
   個位數字相同？ （3分）

   What is the least number of integers that must be chosen from 1, 2, …, 100 to
   ensure that we must be able to find two of the chosen integers with the same unit
   digit? (3 marks)

4. 設 n 為 2007 的正倍數。當 n 乘以 2007 時，積的個位數字是 8。當 n 除以
   2007 時，商的個位數字是甚麼？ （3分）

   Let n be a positive multiple of 2007. When n is multiplied by 2007, the product has
   unit digit is 8. When n is divided by 2007, what is the unit digit of the quotient? (3 marks)

5. 方程 $x^5 = 5x^3 + 36x$ 有多少個不同的實根？ （3分）

   How many different real roots are there for the equation $x^5 = 5x^3 + 36x$? (3 marks)

6. 四人參加一個比賽，他們的得分的中位數和平均數都是 80，而最低分的那人
   得到 64 分。問最高分的那人得到多少分？ （3分）

   Four people joined a competition. Both their mean and median scores are 80, while
   the lowest score among them is 64. What is the highest score among them? (3 marks)

7. 設 [x] 代表不超過 x 的最大整數，例如 [1.1] = 1、[6.9] = 6 和 [5] = 5。求
   $[\log 1] + [\log 2] + [\log 3] + \cdots + [\log 100]$ 的值。 （4分）

   Let [x] denote the greatest integer not exceeding x. For example, [1.1] = 1, [6.9] =
   6 and [5] = 5. Find the value of $[\log 1] + [\log 2] + [\log 3] + \cdots + [\log 100]$. (4 marks)

8. 某校中四甲班有 n 名學生，其中 $1 \le n \le 45$。該班的同學製作了一個題為「四甲班同學最喜愛的科目」的圓形圖。在該圖中，代表「英文科」的扇形的扇形角為 15°。求 n。 （4分）

   In a certain school, there are n students in Secondary 4A, where $1 \le n \le 45$. The class constructed a pie chart entitled 'The favourite subject of 4A students'. The sector representing 'English' has an angle of 15°. Find n. (4 marks)

9. 某主題公園的成人、長者和小童入場券分別價值 295 元、170 元和 210 元。每次購買剛好 24 張入場券時可獲九折優惠。若某人需要 10 張成人、10 張長者和 26 張小童入場券，他最少要付款多少元？ （5分）

   In a theme park, a ticket for adults, elderly and children costs \$295, \$170 and \$210 respectively. The park offers a 10% discount on a one-time purchase of exactly 24 tickets. If someone needs 10 tickets for adults, 10 tickets for elderly and 26 tickets for children, what is the minimum amount (in dollars) that he has to pay? (5 marks)

10. 若某天的「月」和「日」之積是合成數，則那天稱為「好日子」。例如：因為 $1 \times 27 = 27$，而 27 是合成數，所以 1 月 27 日是「好日子」。那麼，2007 年共有多少個「好日子」？ （5分）

    A day is said to be 'good' if the product of the 'month' and the 'day' is a composite number. For example, since $1 \times 27 = 27$ and 27 is a composite number, we say that 27th January is a 'good day'. How many 'good days' are there in the year 2007? (5 marks)

11. 一個半徑為 12 的四分之一圓可以捲成一個容量為 $k\pi$ 的圓錐容器。求最接近 k 的整數。 （5分）

    A quadrant of a circle of radius 12 can be rolled into a conic container of capacity $k\pi$. Find the integer closest to k. (5 marks)

12. 若 $\dfrac{2^3-1}{2^3+1} \times \dfrac{3^3-1}{3^3+1} \times \cdots \times \dfrac{20^3-1}{20^3+1} = \dfrac{a}{b}$，其中 a 和 b 是正整數，且它們的最大公因數為 1，求 $a+b$。 （6分）

    If $\dfrac{2^3-1}{2^3+1} \times \dfrac{3^3-1}{3^3+1} \times \cdots \times \dfrac{20^3-1}{20^3+1} = \dfrac{a}{b}$ where a, b are positive integers with H.C.F. 1, find the value of $a+b$. (6 marks)

13. 在一個三角形中，其中兩條邊的長度相同，而且都比第三條邊短。三邊的長度都是整數，而且都是方程 $x^2 - 127x + k = 0$（其中 k 是常數）的根。求 k 的最小可能值。 （6分）

In a triangle, two sides have equal lengths and are both shorter than the third side. The lengths of the three sides are all integers and all satisfy the equation $x^2 - 127x + k = 0$, where k is a constant. Find the smallest possible value of k. (6 marks)

14. 當小森出生時，他的爸爸 n 歲，其中 $25 \le n \le 40$。從小森出生的那年直至他爸爸 80 歲去世的那年期間（包括首尾兩年），剛好有兩年出現小森爸爸的年齡是小森的年齡的倍數的情況。求 n 的所有可能值之和。 （6分）

When Sam was born, his father was n years old, where $25 \le n \le 40$. From the year Sam was born until his father died at the age of 80 (inclusive), there were exactly two years in which the age of Sam's father is a multiple of the age of Sam. Find the sum of all possible values of n. (6 marks)

15. 設 $[x]$ 代表不超過 x 的最大整數，例如 $[1.1] = 1$、$[6.9] = 6$ 和 $[5] = 5$。若 $[2y] = 5$、$[3y] = 7$，求 $[10y]$ 所有可能值之和。 （6分）

Let $[x]$ denote the greatest integer not exceeding x. For example, $[1.1] = 1$, $[6.9] = 6$ and $[5] = 5$. If $[2y] = 5$ and $[3y] = 7$, find the sum of all possible values of $[10y]$. (6 marks)

16. 某城市規定，市民購物時須繳交相當於貨品原價 5% 的銷售稅。王先生購物後，發現貨品的原價和連稅的價格（以「元」為單位）都是整數。它們的數字都相同，只是數字的次序改變了。已知貨品的原價介乎 1000 元和 4000 元之間，則貨品的原價是多少元？ （7分）

In a city, citizens are required to pay 5% of the original price of goods as sales tax during purchase. After purchase, Mr Wong discovered that both the original price and the price after tax of the goods (in dollars) are integers. They have the same digits but in different orders. Given that the original price of the goods is between 1000 dollars and 4000 dollars, find the original price of the goods in dollars. (7 marks)

17. 若某四位正整數 *abcd* 的四個數字互不相等，並滿足 $a>b$、$b<c$ 和 $c>d$，則稱爲「波浪數」。問共有多少個「波浪數」？　　　　　　　　　（7分）

A four-digit positive integer *abcd* is said to be a 'wave number' if its four digits are pairwise distinct and satisfy $a>b$, $b<c$ and $c>d$. How many 'wave numbers' are there?　　　　　　　　　　　　　　　　　　　　　　　（7 marks）

18. 圖中，*ABC* 是直角三角形，*B* 是直角，而且 $AB=28$、$BC=21$。半圓的直徑位於 *AC* 上，而半圓分別與 *AB* 和 *BC* 相切。小圓則分別與 *AB*、*BC* 和半圓相切。已知小圓的半徑可寫成 $a+b\sqrt{2}$，其中 *a* 和 *b* 是整數。求 a^2+b^2 的值。　　　　　　　　　（7分）
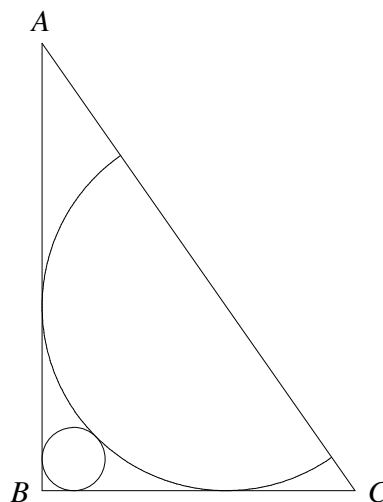
In the figure, ΔABC is right-angled at *B* with $AB=28$ and $BC=21$. The semicircle has its diameter lying on *AC* and is tangent to both *AB* and *BC*. The small circle is tangent to *AB*, *BC* and the semicircle. It is known that the radius of the small circle can be written in the form $a+b\sqrt{2}$, where *a* and *b* are integers. Find the value of a^2+b^2.　　　　　　　　　（7 marks）

19. 在所示的加法中，每個字母代表一個 1 至 9 的不同數字。求 DEFG 所代表的四位數。　　　　　（7分）

In the addition shown, each letter represents a different integer from 1 to 9. Find the four-digit number represented by DEFG.　　　　　（7 marks）

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    A B C
    A C B
    B A C
    B C A
    C A B
+   C B A
  D E F G
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20. 某人寫下一個三位數 N，然後另外五人分別對 N 的值作出了一些猜測。

甲說：「N 能被 27 整除。」

乙說：「N 能被 11 整除。」

丙說：「N 的數字之和是 15。」

丁說：「N 是一個平方數。」

戊說：「N 是 648000 的因數。」

已知五人當中有三人猜對了，兩人猜錯了。求 N。　　　　　　　　（8分）

Someone wrote down a three-digit number N and five others made some guesses on the value of N.

A said, 'N is divisible by 27.'

B said, 'N is divisible by 11.'

C said, 'The sum of the digits of N is 15.'

D said, 'N is a square number.'

E said, 'N is a factor of 648000.'

It is known that among the five guesses, three were correct and two were wrong. Find N. (8 marks)

全卷完

END OF PAPER