第六屆培正數學邀請賽

6th Pui Ching Invitational Mathematics Competition

初賽（中三組）

Heat Event (Secondary 3)

時限：1 小時 15 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

1. 本卷共設 20 題，總分爲 100 分。

   There are 20 questions in this paper and the total score is 100.

2. 除特別指明外，本卷內的所有數均爲十進制。

   Unless otherwise stated, all numbers in this paper are in decimal system.

3. 所有答案皆是 0 至 9999 之間的整數（包括 0 和 9999）。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

   All answers are integers between 0 and 9999 (including 0 and 9999). Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

4. 不得使用計算機。

   The use of calculators is not allowed.

5. 本卷的附圖不一定依比例繪成。

   The diagrams in this paper are not necessarily drawn to scale.

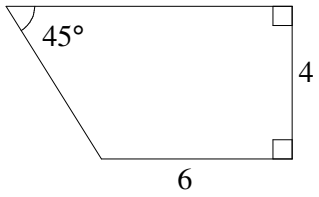
1. 小斌把所有小於 1000000、個位數字不是 0 而且是 2007 的倍數的正整數乘起來。乘積的最後四位數字是甚麼？ （3 分）

Purple multiplied together all positive integers less than 1000000 which are multiples of 2007 with unit digit non-zero. What are the last four digits of the product? (3 marks)

2. 設 n 爲大於 1 的整數。小均在計算 n^{n^n} 時，誤把算式看成 $(n^n)^n$，卻意外地得到正確答案。求 n。 （3 分）

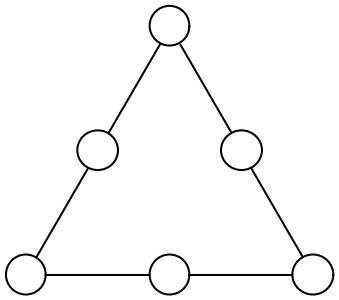
Let n be an integer greater than 1. When computing n^{n^n}, Wendy wrongly regarded the expression as $(n^n)^n$, but accidentally got the correct answer. Find n. (3 marks)

3. 求圖中的四邊形的面積。（如有需要，答案準確至最接近整數。） （3 分）



Find the area of the quadrilateral in the figure. (Round off the answer to the nearest integer if necessary.) (3 marks)

4. 現要在圖中的六個圓中分別填上 1 至 6，並計算每條邊上三個圓內的數字之和。這三個數字之和加起來的最小值是甚麼？ （3 分）



The numbers 1 to 6 are to be filled into the six circles in the figure. The sum of the three numbers in the three circles on each side is then computed and the three sums are added up. What is the smallest possible value of the result obtained? (3 marks)

5. 小琪帶了共 n 個一元、二元、五元和十元硬幣去購物。她打算買一件不超過 100 元，而且價格（以「元」爲單位）是整數的貨品，並肯定可以毋須找贖。求 n 的最小可能值。 （4 分）

Kiki brings n coins, each of denomination \$1, \$2, \$5 or \$10, for shopping. She plans to buy an article whose price (in dollars) is an integer and which does not exceed \$100. She is sure that she can pay the exact amount of the article. Find the smallest possible value of n. (4 marks)

6. 一個 2007 邊形中，最多可以有多少隻內角大於 270°？ （4 分）

At most how many interior angles of a 2007-sided polygon may be larger than 270°? (4 marks)

7. 一名男孩打算購買一張某主題公園的小童入場券，價值 210 元，並且必須繳付準確的數目。如果該男孩有 20 個五元硬幣和 20 個十元硬幣，他共有多少種不同的方法付款？ （4 分）

A boy wants to buy a child ticket of a theme park which costs $210 and which accepts only exact amounts. If he has twenty $5 coins and twenty $10 coins, in how many different ways can he pay for the tickets? (4 marks)

8. 某國家每月徵收薪俸稅，所有月入 4000 元以上的市民，在扣除 4000 元免稅額後，均要繳納餘額的 20% 作爲薪俸稅。陳先生本月加薪 10% 後，所繳納的薪俸稅比上月高了 20%。問陳先生本月繳納的薪俸稅是多少元？ （4 分）

In a country, salaries tax is to be paid on a monthly basis and applies to all citizens with a monthly salary exceeding $4000. The salaries tax is 20% of the monthly income after deduction of an allowance of $4000. This month, Mr Chan has his income increased by 10%, and the amount of salaries tax he has to pay is 20% greater than in the previous month. What is the amount (in dollars) of salaries tax that Mr Chan has to pay this month? (4 marks)

9. 設 k 爲正整數。若方程組 $\begin{cases} 2x+3y-6=0 \\ 3x-2y+k=0 \end{cases}$ 中 x 和 y 的解都是整數，求 k 的最小可能值。 （5 分）

Let k be a positive integer. If the system of equations $\begin{cases} 2x+3y-6=0 \\ 3x-2y+k=0 \end{cases}$ has integer solutions in x and y, find the smallest possible value of k. (5 marks)

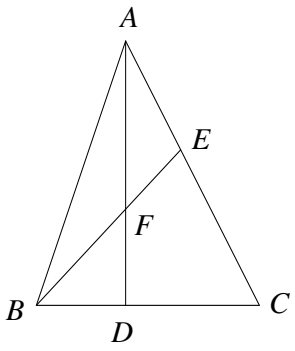
10. 從 1、2、…、100 中，最少要選出多少個整數，才能確保當中必定有兩個之和的個位數字是 7？ （5 分）

What is the least number of integers that must be chosen from 1, 2, …, 100 to ensure that we must be able to find two of the chosen integers whose sum has unit digit 7? (5 marks)

11. 某校中三甲班有 n 名學生，其中 $1 \le n \le 45$ 。該班的同學製作了一個題爲「三甲班同學最喜愛的科目」的圓形圖。在該圖中，代表「數學科」的扇形的扇形角爲 285°。求 n。（5 分）

In a certain school, there are n students in Secondary 3A, where $1 \le n \le 45$. The class constructed a pie chart entitled 'The favourite subject of 3A students'. The sector representing 'Mathematics' has an angle of 285°. Find n. (5 marks)

12. 圖中，AFD、BFE、BDC 和 AEC 都是直線，而且 $BD : DC = 2 : 3$ 、 $AE : EC = 4 : 5$ 、 $AF : FD = x : y$ ，其中 x 和 y 都是整數，且它們的最大公因數爲 1。求 xy。（5 分）



In the figure, AFD, BFE, BDC and AEC are straight lines with $BD : DC = 2 : 3$, $AE : EC = 4 : 5$ and $AF : FD = x : y$, where x, y are integers with H.C.F. 1. Find xy. (5 marks)

13. 設 $[x]$ 代表不超過 x 的最大整數，例如 [1.1] = 1、[6.9] = 6 和 [5] = 5。若 $[2y] = 5$ 、 $[3y] = 7$ ，求 $[10y]$ 所有可能值之和。（6 分）

Let $[x]$ denote the greatest integer not exceeding x. For example, [1.1] = 1, [6.9] = 6 and [5] = 5. If $[2y] = 5$ and $[3y] = 7$, find the sum of all possible values of $[10y]$. (6 marks)

14. 麗娜和美芳住在某條道路的兩端。某天正午，她們同時駕車向對方駛去。她們原定會在下午 2 時相遇。在下午 1 時，麗娜發現自己忘了帶東西，於是她便立即駛回家中。當她抵達家中時便立即重新起程。最後她們在下午 2 時 40 分相遇。假設她們的速率一直不變而麗娜的速率是 14km/h，這條路長多少公里？（6 分）

Laura and Molly live on two ends of a road. One day, they started driving towards each other at noon. Originally, they should meet at 2:00 pm. However, Laura found that she left something at home at 1:00 pm, so she drove back to home at that time. She restarted her journey as soon as she was back to home. Finally they met at 2:40 pm. Assuming that their speeds remained unchanged throughout the journey and Laura's speed was 14km/h, what is the length of the road (in km)? (6 marks)

15. 若某天的「月」和「日」之積是合成數，則那天稱爲「好日子」。例如：因爲 $1\times27=27$，而 27 是合成數，所以 1 月 27 日是「好日子」。那麼，最多可以有連續多少天的「好日子」？（6 分）

A day is said to be 'good' if the product of the 'month' and the 'day' is a composite number. For example, since $1\times27=27$ and 27 is a composite number, we say that 27th January is a 'good day'. What is the maximum possible number of consecutive 'good days'? (6 marks)

16. 如圖所示，某個國家的地形是正方形，並分成四個區域。該國有一些公園，每個都位於一個區域內或橫跨兩個相鄰（即圖中有公共邊）的區域。每個區域中的整數表示完全或部分位於該區域內的公園的數目。問該國家最少有多少個公園？（6 分）

13	25
24	30

A country is in the shape of a square and is divided into four regions as shown. There are some parks in the country and each park lies in one region or across two neighbouring regions (i.e. regions with a common edge in the figure). The integer in each region denotes the number of parks which lie wholly or partly in the region. What is the minimum total number of parks in the country? (6 marks)

17. 一個凸多面體的每個面都是五邊形。若它有 20 個頂點，它有多少條邊？（6 分）

A convex polyhedron has 20 vertices and each of its faces is a pentagon. How many edges does it have? (6 marks)

18. 已知 A、B、C、x、y、z 是大於 1 的正整數，其中 $A<B<C$ 而 $xyz=105$。在 1 和 A（包括 1 和 A，下同）之間有 x 個質數，在 $A+1$ 和 B 之間有 y 個質數，在 $B+1$ 和 C 之間有 z 個質數。那麼 (A, B, C) 有多少組不同的可能值？（7 分）

Given A, B, C, x, y, z are integers greater than 1 such that $A<B<C$ and $xyz=105$. There are x prime numbers between 1 and A (inclusive, same for below), y prime numbers between $A+1$ and B, and z prime numbers between $B+1$ and C. How many sets of possible values of (A, B, C) are there? (7 marks)

19. 在所示的加法中，每個字母代表一個 1 至 9 的不同數字。求 DEFG 所代表的四位數。（7 分）

In the addition shown, each letter represents a different integer from 1 to 9. Find the four-digit number represented by DEFG. (7 marks)

$$\begin{array}{rcccc} & A & B & C \\ & A & C & B \\ & B & A & C \\ & B & C & A \\ & C & A & B \\ + & C & B & A \\ \hline D & E & F & G \end{array}$$

20. 某人寫下一個三位數 N，然後另外五人分別對 N 的值作出了一些猜測。

甲說：「N 能被 27 整除。」
乙說：「N 能被 11 整除。」
丙說：「N 的數字之和是 15。」
丁說：「N 是一個平方數。」
戊說：「N 是 648000 的因數。」

已知五人當中有三人猜對了，兩人猜錯了。求 N。（8 分）

Someone wrote down a three-digit number N and five others made some guesses on the value of N.

A said, 'N is divisible by 27.'
B said, 'N is divisible by 11.'
C said, 'The sum of the digits of N is 15.'
D said, 'N is a square number.'
E said, 'N is a factor of 648000.'

It is known that among the five guesses, three were correct and two were wrong. Find N. (8 marks)

全卷完

END OF PAPER