第六屆培正數學邀請賽

6th Pui Ching Invitational Mathematics Competition

初賽（中二組）

Heat Event (Secondary 2)

時限：1 小時 15 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

1. 本卷共設 20 題，總分爲 100 分。

   There are 20 questions in this paper and the total score is 100.

2. 除特別指明外，本卷內的所有數均爲十進制。

   Unless otherwise stated, all numbers in this paper are in decimal system.

3. 所有答案皆是 0 至 9999 之間的整數（包括 0 和 9999）。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

   All answers are integers between 0 and 9999 (including 0 and 9999). Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

4. 不得使用計算機。

   The use of calculators is not allowed.

5. 本卷的附圖不一定依比例繪成。

   The diagrams in this paper are not necessarily drawn to scale.

1. 某凸 n 邊形的內角和與外角和相等。求 n。（3 分）

A convex n-sided polygon has its sum of interior angles equal to its sum of exterior angles. Find n. (3 marks)

2. 一個正方形中心與各頂點的距離都是 20。求這正方形的面積。（3 分）

In a square, the distance from the centre to each vertex is 20. Find the area of the square. (3 marks)

3. 某學校有 2007 名學生，他們分成 3 人或 5 人一組，使得每人均屬於剛好一組。這些學生最少分成了多少組？（3 分）

In a school, there are 2007 students, and they are divided into groups of either 3 or 5 students so that each student belongs to exactly one group. What is the minimum number of groups? (3 marks)

4. 某城市的鐵路共有 11 個車站，它們圍成一個圈以順時針方向循環線運作。包括上落客時間在內，列車從一個車站到下一個車站需時 7 分鐘。若要維持列車的班次在 3 分鐘一班，則最少要派出多少部列車行走？（4 分）

The railway of a city consists of 11 stations arranged in a loop and operates in a circular route in the clockwise direction. It takes 7 minutes for the train to travel from one station to the next, including the time for the boarding and alighting. If the frequency of trains has to be maintained at 3-minute intervals, what is the minimum number of trains needed? (4 marks)

5. 小莉到百貨公司購物。她買了 10 件貨品，每件的價格（以「元」爲單位）都是整數。小莉爲了估算貨品的總值，把每件貨品的價格四捨五入至最接近 10 元然後加起來，並得到 1230 元的估算值。問貨品的確實總值最高是多少元？（4 分）

Lily went shopping in a department store. She bought 10 items, and the price (in dollars) of each of them is an integer. To estimate the total price of the items, Lily rounded off the price of each item to the nearest $10 and added them up to get the estimated sum of $1230. What is the maximum possible value (in dollars) of the actual sum? (4 marks)

6. 在一次數學競賽中，某道題目要求參賽者計算某個正方體的體積的數值。其中一名參賽者把「體積」錯看成「表面積」，卻意外地得到正確的答案。該題的正確答案是多少？ （4 分）

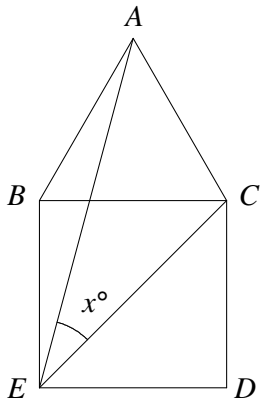
In a mathematical competition, a question asks for the numerical value of the volume of a cube. A contestant misread 'volume' as 'surface area', but surprisingly got the correct answer. What is the correct answer to the question? (4 marks)

7. 有多少個兩位正整數的數字之和是平方數？ （4 分）

How many two-digit positive integers have the property that the sum of its digits is a square number? (4 marks)

8. 圖中，ABC 是等邊三角形，而 $BCDE$ 是正方形。若 $\angle AEC = x°$，求 x。 （4 分）

In the figure, ABC is an equilateral triangle while $BCDE$ is a square. If $\angle AEC = x°$, find x. (4 marks)



9. 現有 5 塊邊長爲 1 的正方形咭片和 5 塊邊長爲 1 的等邊三角形咭片。若把這 10 塊咭片拼成一個多邊形，咭片之間不許互相重疊，那麼拼出的多邊形最少有幾條邊？ （5 分）

There are 5 square cardboards of side length 1 and 5 cardboards in the shape of equilateral triangles of side length 1. These 10 cardboards are to be put together to form a polygon. If overlapping of cardboards is not allowed, what is the least number of sides of the resulting polygon? (5 marks)

10. 若 $2007^n + 2006^n + 2005^n + 2004^n + 2003^n + 2002^n$ 的個位數字是 7，其中 n 是小於 2007 的整數，求 n 的最大可能值。 （5 分）

If the unit digit of $2007^n + 2006^n + 2005^n + 2004^n + 2003^n + 2002^n$ is 7, where n is an integer less than 2007, find the greatest possible value of n. (5 marks)

11. 當 $20072007^2 - 20062006^2$ 除以 10000 時，餘數是多少？ （5 分）

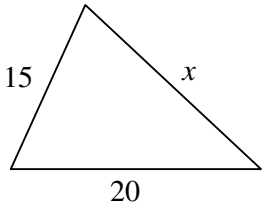
What is the remainder when $20072007^2 - 20062006^2$ is divided by 10000? (5 marks)

12. 在培正數學邀請賽初賽中，每題的佔分都是 2 至 10 之間的整數（包括 2 和 10），滿分爲 100。某校派出了 n 名學生參加這比賽，而他們各人的得分的數字之和互不相同。求 n 的最大可能值。 （6 分）

In the Heat Events of the Pui Ching Invitational Mathematics Competition, the score carried by each question is an integer between 2 and 10 (inclusive), and the full score is 100. A school sends n students for the competition, and it turns out that the sums of digits of the scores of the students are pairwise distinct. Find the greatest possible value of n. (6 marks)

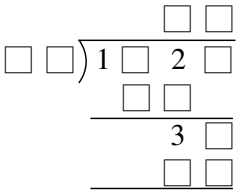
13. 附圖是一個面積爲 120 的銳角三角形。求最接近 x 的整數。 （6 分）

The figure shows an acute-angled triangle with area 120. Find the integer closest to x. (6 marks)



14. 圖中顯示一條除式，但當中有些數字留空了。求商（即最頂一行的兩位數）的所有可能值之和。 （6 分）

The figure shows a division, but some digits are left out. Find the sum of all possible values of the quotient (i.e. the two-digit number at the top row). (6 marks)



15. 2006 位學生圍成一個圈坐著，之後老師派給每人一張黃色、紅色或藍色的咭片。每人都只能看見自己和身旁兩人的咭片。接著，老師問他們看見的咭片的顏色時，每人都回答他們看見黃色、紅色和藍色的咭片各一張。他們當中最少有多少人說了謊？ （6 分）

2006 students sit in a circle. The teacher then gives each of them a card which must be yellow, red or blue. Every student can only see his own card and the cards of his two neighbours. When the teacher asks them about the colours of the cards they can see, all students reply that they see a yellow card, a red card and a blue card. At least how many students have lied? (6 marks)

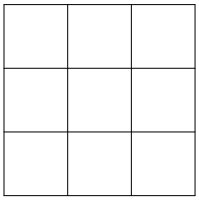
16. 小明和小芬都會按每天的日期中的「日」決定當天穿甚麼顏色的衣服。小明會把日期中的「日」除以 8，如果餘數是偶數便穿紅色衣服，否則穿黃色衣服。小芬則會把日期中的「日」除以 6，如果餘數是 0 或 3 便穿綠色衣服，餘數是 1 或 4 便穿黃色衣服，餘數是 2 或 5 便穿藍色衣服。例如：因爲 27 除以 8 和 6 時餘數都是 3，所以在 1 月 27 日小明會穿黃色衣服，小芬則會穿綠色衣服。在 2007 年中，小明和小芬有多少天穿上相同顏色的衣服？ （6 分）

Mike and Fanny both determine the colour of the clothes to wear according to the 'day' of the date. Mike will divide the 'day' by 8 and wear red clothes if the remainder is even, and wear yellow clothes if otherwise. Fanny will divide the 'day' by 6 and wear green clothes if the remainder is 0 or 3, yellow clothes if the remainder is 1 or 4, and blue clothes if the remainder is 2 or 5. For example, since 27 leaves a remainder of 3 when divided by both 8 and 6, Mike wears yellow clothes on 27th January while Fanny wears green clothes. For how many days in the year 2007 will Mike and Fanny wear clothes of the same colour? (6 marks)

17. 某城市制訂了一條法例，有效期爲 2006 年 5 月 1 日至 2009 年 5 月 1 日（包括首尾兩天）。法例規定未滿 16 歲的人士一律不得進入遊戲機中心，而所有遊戲機中心必須在入口處展示「以下日期以後出生的人士不得進入」的告示，告示的下方則以「年年年年／月月／日日」的八位數字形式展示日期。例如：在 2007 年 1 月 27 日，所展示的日期爲「19910127」。每個數字都必須以咭紙製成。若每張數字咭只可代表一個數字，那麼每所遊戲機中心最少要製作多少張數字咭才可確保法例生效期間的每天都能展示出所需的日期？ （6 分）

A city has enacted a law which is effective from 1st May 2006 to 1st May 2009 (inclusive). The law prohibits people aged under 16 to enter game centres. All game centres are required to show at their entrance a sign stating 'People born after the following date are not allowed to enter', followed by a 8-digit date represented by the 'YYYY/MM/DD' format. For instance, on 27th January 2007, the date shown will be '19910127'. Each digit must be shown by a cardboard. If each cardboard can only represent one digit, what is the least number of cardboards that each game centre must make to ensure that the required date can be correctly shown throughout the period of the law? (6 marks)

18. 小桐把 1 至 9 各數填進圖中的 3×3 方格表內，使得每個填上奇數的方格的所有相鄰方格內都填上偶數，而每個填上偶數的方格的所有相鄰方格都填上奇數。問共有多少種不同的方法填數字？（6 分）



Stone fills the integers 1 to 9 into the 3×3 grid shown in the figure in a way such that for every cell which is filled with an odd integer, all its adjacent cells are filled with even integers, and for every cell filled with an even integer, all its adjacent cells are filled with odd integers. In how many different ways can Stone fill in the numbers? (6 marks)

19. 一間薄餅店提供外賣速遞服務。假設每名顧客購買的食品總值都不超過 1000 元，而且都是 0.1 元的倍數，那麼送貨員只要帶備 m 張紙幣和 n 個硬幣，便肯定能夠準確地找贖。（可用的紙幣和硬幣則只有 1000 元、500 元、100 元、50 元、20 元、10 元、5 元、2 元、1 元、0.5 元、0.2 元和 0.1 元共 12 種。）求 $m+n$ 的最小可能值。（7 分）

A pizza café offers delivery service. Assuming that the total value of the food ordered by each customer does not exceed $1000 and must be a multiple of $0.1, then the delivery staff can ensure accurate change to be given by bringing along m notes and n coins. (There are 12 types of notes and coins, of denominations $1000, $500, $100, $50, $20, $10, $5, $2, $1, $0.5, $0.2 and $0.1). Find the smallest possible value of $m+n$. (7 marks)

20. 一部郵票售賣機售賣面值 1.4 元、1.8 元、2.4 元和 3 元的郵票。若小明需要剛好 12 元的郵票，他所買的郵票可以有多少種不同的面值組合？（7 分）

A vending machine sells stamps of the values $1.4, $1.8, $2.4 and $3. If Mike needs stamps worth exactly $12, how many different combinations of values are possible for the stamps he buys? (7 marks)

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END OF PAPER